Cochrane’s defence of the fiscal theory of the price level – clarification and critique

Colin Rogers
School of Economics
University of Adelaide

Abstract
Cochrane (2005) employs a well specified Walrasian general equilibrium model to defend the fiscal theory of the price level against two forms of criticism; that: (1) it implies the violation of Walras’s Law and, (2) the completely cashless, frictionless version of the model cannot determine the price level. Of the two criticisms only the second is substantive. This paper explains why no version of Cochrane’s model offers a theory of the price level. The model can determine the consumption good value of an asset and a numeraire denominated price level. A numeraire denominated price level is of no theoretical significance.

Key Words: Frictionless; Cashless; fiscal theory of price level; Walras’s Law.

JEL classification: E31; E42.

Corresponding author: Colin Rogers, School of Economics, University of Adelaide, North Terrace, Adelaide, South Australia, 5005. Tel: (08) 83034394 fax: 8223 1460. colin.rogers@adelaide.edu.au
Cochrane’s defence of the fiscal theory of the price level – clarification and critique

Colin Rogers
School of Economics
University of Adelaide

Revised February 9, 2006

Introduction

The fiscal theory of the price level (FTPL) is the idea that the government’s intertemporal budget constraint can be employed to determine the general price level and that this relationship applies even in a cashless or moneyless, frictionless world where the intertemporal budget constraint is treated as a pricing kernel for the public debt. In a recent paper Cochrane (2005) presents a defence of the FTPL against the theoretical critique by Buiter (1998, 1999, 2002, 2004) and McCallum (2003, 2004). McCallum argues that the FTPL is misleading and not applicable to real economies while Buiter (1999, p. 1) represents the most strident statement of the criticism when he states:

“It is not common for an entire scholarly literature to be based on a fallacy, that is ‘on faulty reasoning; misleading or unsound argument’. The recently revived ‘fiscal theory of the price level’ is an example of a research programme that is fatally flawed, conceptually and logically.”

Buiter’s criticism is based on what he considers to be an economic misspecification – the proposition that the government’s intertemporal budget constraint need only be satisfied in equilibrium. In a Walrasian general equilibrium model that is equivalent to the claim that Walras’s law applies only in equilibrium. Buiter (1999, p. 1, 2002, p. 478) goes on to argue that this economic misspecification is the cause of conceptual anomalies such as the apparent ability to price money in a world without money.
In this paper I explain why the question of Walras’s Law is largely a red herring that has no direct implications for the FTPL. In a well-specified Walrasian general equilibrium model, as employed by Cochrane (2005), Walras’s Law applies in all states of the model and all budget constraints are always satisfied. That effectively dispenses with Buiter’s argument that the FTPL implies that the government budget constraint only holds in equilibrium. Despite this conclusion it does not absolve the FTPL of the charge that it produces a rash of contradictions and anomalies. The reason is that those anomalies originate with the frictionless model that Cochrane uses to support the FTPL and do not derive from the alleged failure of the model to satisfy Walras's Law in disequilibrium as claimed by Buiter (2002). The anomalies generated by the FTPL are examples of theoretical and conceptual hurdles that result from the application of Walrasian perfect or efficient barter general equilibrium models to questions of monetary and fiscal theory1 as for example in Woodford (1998, 2003).

The basis of Cochrane’s (2005) defence of the FTPL is an analogy between the pricing of stock and the pricing of government bonds: the belief that nominal debt is mathematically identical to stock, Cochrane (2005, p. 526). Cochrane (2005, p. 504, emphasis added) applies that idea in the context of a:

“…perfectly standard and well-specified Walrasian economic model, one in which the government has no special status, and one in which all budget constraints are satisfied at both equilibrium and disequilibrium price levels. It may or may not apply to a given time and place but it is at least a theoretically coherent possibility”.

In this paper I argue there are serious logical and conceptual hurdles raised by Cochrane’s analysis that undermine its ability to provide a theoretically coherent

---

1. As Lucas (1984, p. 32) explained, these perfect barter or frictionless models appeal because it was thought that the successful empirical application of these frictionless models in financial economics could be repeated in monetary economics. However, Cochrane does not offer an empirical defence of the frictionless barter model but aims only to present a theoretically coherent defence of the FTPL.
account of the price level in the frictionless, well-specified Walrasian general equilibrium system. In that context the conceptual hurdles noted by Buiter and McCallum are not merely semantic (e.g., what is meant by the ‘price level’, ‘cashless’ or ‘frictionless’) but raise serious doubts about the logical and theoretical coherence of Cochrane’s model of the FTPL. They also raise fundamental questions about the use of the frictionless model as a basis for monetary and fiscal theory.

The conceptual hurdles originate with, and are inherent in, the version of the Lucas tree model with a cash-in-advance constraint employed by Cochrane, but are highlighted by his extension of the model to a moneyless, frictionless state. The model has all the features of a Walrasian general equilibrium system. That is, budget constraints are always satisfied and Walras’s Law holds. However, it is another feature of the model that effectively undermines Cochrane’s use of the model as the basis for the FTPL. Fiat or inconvertible money only exists, as Sargent (1987, chapter 5) points out, because restrictions are placed on household exchange patterns. Without these restrictions fiat money has zero exchange value in such models. This is an example of a problem first noticed by Hahn (1965, 1973a, 1973b, 1982) with respect to Patinkin’s model but it applies to Walrasian general equilibrium systems in general (see Sargent 1987, chapter 3). Models with this feature are best described as models of efficient or perfect barter (as opposed to real world barter) and when money is incorporated into the model, in the form of a cash-in-advance (CIA) constraint or in the utility function, it supports an inferior set of equilibria as a result of the restrictions it places on exchanges. This feature of Walrasian general equilibrium systems puzzled monetary theorists in the past but is now widely understood\(^2\).

\(^2\) Clower (1984, p. 267) was puzzled by the fact that money appeared to be a welfare reducing friction in his model: “...the choice alternatives confronting households were more restrictive in a money than in a barter economy, which meant that monetary exchange is less efficient that barter exchange.
The relevance of all this to Cochrane’s defence of the FTPL arises directly from his application of a well specified Walrasian general equilibrium system in both its monetary and non-monetary and frictionless versions. For Cochrane (2005, p.503):

“[F]rictionless competitive models are the benchmark, the foundation upon which we add interesting frictions. Yet monetary economics has so far crucially relied on a big friction at the short end of the yield curve in order to even start talking about a price level”.

Cochrane’s objective is to eliminate the ‘big friction’ at the short end of the yield curve, introduced by the quantity equation or a CIA constraint, to produce his frictionless benchmark model, and to employ that model to determine the price level. But by eliminating the ‘big friction’ Cochrane creates the conceptual and logical anomalies noted by Buiter (1999, 2002, 2004) and McCallum (2003, 2004). The conceptual difficulties arise because by eliminating the medium of exchange function of money Cochrane is forced to change the concept of money, and along with it the concept of the price level, when moving between the different versions of the model.

Traditional quantity theory interpretations of the model include fiat or inconvertible money and impose constraints on households to ‘force’ them to hold intrinsically worthless money. The price level in that context is the inverse of the purchasing power of money and is determined by the quantity equation or defined in terms of prices denominated in the medium of exchange. Moving to the frictionless version of the model eliminates all the constraints that give fiat money positive exchange value contrary to both common sense and two hundred years of conventional wisdom. Something obviously was wrong. But what? What is wrong is that money has been added to a model where it is not needed – a model of perfect barter where a medium of exchange is redundant. Walrasian, Arrow-Debreu models based on a time-0 auction have no role for fiat money as a medium of exchange, as Hahn (1982, p.1) noted. Hence imposing the medium of exchange function reduces money to a friction.
and money is reinterpreted as an income-earning asset. The price level can no longer be determined by the quantity equation and that concept is abandoned, and replaced by the concept of a numeraire price level—the price level defined in terms of the numeraire prices of all commodities. It is here that the conceptual problems arise because numeraire prices are arbitrary and have no economic significance. Consequently any price level defined in terms of them has a similar interpretation. This, in essence is the cause of the objections to the FTPL by Buiter and others—*the FTPL is not a theory at all but an arbitrary definition of no economic significance*. To make the case the remainder of the paper proceeds as follows.

Section II outlines the three states of the model identified by Cochrane—the monetary, cashless and completely cashless states. I suggest a clarification of Cochrane’s terminology: that the terms cashless and frictionless are of analytical significance only in the context of the completely cashless and frictionless model when all frictions, including the CIA constraint—Cochrane’s ‘big friction’—have been eliminated. From this perspective there are only two analytically distinct versions of Cochrane’s model—the version with the CIA constraint (the monetary and cashless versions) and the completely cashless and frictionless version where the CIA constraint has been removed. The cashless version is analytically similar to the monetary version because both retain the big friction on intra day trades.

Section III then interprets Cochrane’s model. First, it applies Walras’s Law to illustrate why the monetary and cashless versions cannot differentiate between the FTPL and the quantity theory of the price level (QTPL). Second, and more importantly, Patinkin’s (1965) distinction between *accounting prices* and *money prices* is employed to show that the completely cashless and frictionless model
determines only the real relative prices of commodities and the accounting or
numeraire prices are entirely arbitrary. Defining the price level in terms of numeraire
prices is possible but the price level defined in this way is not determined by the
model- it is as arbitrary as the numeraire prices of goods and has no analytical
significance. This explains Buiter’s characterisation of the FTPL as an attempt to
determine the price of a non-existent entity like phlogiston. Finally, Section III briefly
outlines how the concept of money changes- from fiat money to asset money- as the
analysis moves between the monetary and completely cashless, frictionless versions
of the model.

Section IV concludes that the objections to the FTPL raised by Buiter and McCallum
are substantive, and not merely semantic, when that theory is presented in the context
of a frictionless Walrasian general equilibrium model. Exponents of the FTPL need to
look elsewhere for theoretical support.

II    Cochrane’s monetary, ‘cashless’, and completely cashless worlds

Cochrane begins by stressing the analogy between the pricing of equity (stock) and
the pricing of government bonds. The two relationships are listed as expressions (1)
and (2)\(^3\).

\[
\frac{\text{Nominal value of shares}}{\text{Price level}} = \text{Expected present value of dividends.} \quad (1)
\]

\[
\frac{\text{Nominal Government debt}}{\text{Price level}} = \text{Expected present value of primary surplus.} \quad (2)
\]

Equation (2) is an application of the idea that the real value of stock, as the expected
present value of the dividend stream, can be applied directly to determine the real

\(^3\) Equation numbers follow Cochrane (2005) for ease of comparison. I have amended expression (1) to include the
nominal value of shares in the numerator instead of the number of shares as in Cochrane. Both expressions then
have real (purchasing power) value on the LHS.
value of government debt as the expected present value of the primary surplus. This is the basis of the idea that nominal debt is mathematically identical to equity. That may or may not be so. But even if we accept the mathematical equivalence of expressions (1) and (2) there are several conceptual hurdles that need to be overcome before we can accept Cochrane’s FTPL.

These hurdles arise because Cochrane seeks to move seamlessly between various states of the model without realising that the concepts of money and the price level change across different states of the model. The conceptual problems arise with the use of the completely cashless and frictionless model as the benchmark to which interesting frictions, like money, can be added. Cochrane (2005, p. 503) explains his strategy as follows.

“More importantly, a frictionless model can provide a useful benchmark for more complex and realistic analyses with frictions. Throughout economics, frictionless competitive models are the benchmark, the foundation upon which we add interesting frictions. Yet monetary economics has so far crucially relied on a big friction at the short end of the yield curve in order to even start talking about the price level”.

To see how the conceptual hurdles arise we begin with the monetary version of the well-specified Walrasian general equilibrium system.

Cochrane’s monetary model

Cochrane (2005) employs a Lucas tree model with a cash-in-advance constraint as presented by Sargent (1987, chapter 5). Sargent (1987, Table 5.1, p. 158) provides a detailed outline of the trading pattern and the restrictions on households needed to justify a role for intrinsically worthless fiat money in the model. Without these restrictions fiat money would have zero exchange value (Sargent 1987, chapters 3, 5). The model has three components or Walrasian markets (to use traditional terminology) - goods, assets (bonds/stock) and money - in an intertemporal setting
with trading days extending out to an infinite horizon. Three types of securities are traded in Sargent’s model, shares in trees, government issued currency and one period state contingent claims to currency. But only two securities, currency and nominal government bonds appear in budget constraints as Cochrane notes that shares in trees will be in zero net supply and consequently have no effect on the equilibrium solution. They are, however, important to his application of the analogy between stock and bonds and the mathematically identity in pricing the two assets.

There are many identical households and a government. Each household owns a tree that produces fruit (dividends) but households are precluded from consuming their own fruit\(^4\). Identical households maximise a standard utility function, \(\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)\), have shopper and worker characteristics, and enter period \(t\) with money balances \(M_{i-1}\) and one period nominal discount bonds with face value \(B_{i-1}\). The government chooses a state-contingent sequence of one-period nominal debt, money and primary surpluses, \(\{B^s_t, M^s_t, s_t\}\) and each are random variables. The model starts at \(t = 0\) so \(B_{i-1}\) and \(M_{i-1}\) are fixed (they may both be set to zero).

In the asset market the household sells bonds for currency, pays lump sum taxes, \(p_t s_t\), buys new bonds, \(B_t\) and leaves with money, \(M^d_t\). In the goods market the household receives an endowment, \(e_t\) of the consumption good (fruit from the tree) and cannot consume its own endowment so must trade with other identical households. The household has worker and shopper characteristics. The shopper uses the money from the asset market, \(M^d_t\) to buy goods, \(c_t\) subject to a cash-in-advance constraint, \(^4\) This eliminates the autarky solution.
where \( v = \text{velocity} \) in a quantity equation and \( p_t \) is the price level. Cochrane initially sets \( v = 1 \). The worker sells the endowment \( e_t \) for money and gets cash \( p_t e_t \) in return. In the monetary state of the model, the worker and shopper ‘go home’ and eat \( c_t \). Also they must hold, overnight, any money balances not converted into consumption goods by the shopper, the balance would be; \( M^d_t - p_t c_t \), plus the money balances earned by the worker, and equal to \( p_t e_t \). Hence we have:

\[
M_t = M^d_t + p_t(e_t - c_t)
\]

(6)

To derive the household’s budget constraints Cochrane notes that they can trade arbitrary contingent claims (shares in trees) in the asset market and so the price of a one period state contingent share at time \( t \) is given by;

\[
Q_t = p_t E_t \left( \frac{1}{1 + p_{t+1}} \right)
\]

(7)

The expression \( m_{t,t+1} \) is a real stochastic discount factor or pricing kernel. As noted above, since all households are identical, claims provided by households (shares in trees) are in zero net supply, so do not appear in the household budget constraints and do not influence equilibrium prices or allocations. Only government bonds appear in the household’s period-by-period budget constraint;

\[
B_{t-1} + M_{t-1} + p_t(e_t - c_t) = Q_t B_t + M_t + p_t s_t
\]

(9)

Prohibiting the households from issuing money (to prevent arbitrage against interest-bearing bonds) and applying a transversality condition produces what Cochrane calls the present value budget constraint:
\[ \frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} m_{t+j} (s_{t+j} + c_{t+j} - e_{t+j}) \]  

(11)

In equilibrium the goods, money and asset markets are described by the following three equations in the case of the monetary version of the model.

\[ \beta_j \frac{u'(e_{t+j})}{u'(e_t)} = m_{t+j} \]

(12)

\[ M_{t+j} = p_t c_t = p_t e_t \]

(14)

\[ \frac{B_{t-1}}{p_t} = \sum_{j=0}^{\infty} E_t \left[ m_{t+j} (s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{p_t}) \right] \]

(15)

An equivalent version of (15) is written as;

\[ \frac{B_{t-1} + M_{t-1}}{p_t} = \sum_{j=0}^{\infty} E_t \left[ m_{t+j} (s_{t+j} + \frac{r_{t+j}^f}{1 + r_{t+j}^f} \frac{M_{t+j}}{p_{t+j}}) \right] \]

(16)

where \( r_{t}^f \) is the one period nominal rate of interest such that \( Q = \frac{1}{1 + r_{t}^f} \).

Cochrane introduces one change to Sargent’s restrictions to relieve what he calls a friction that arises from the need for households to hold money balances overnight, thereby foregoing potential interest income. To that end the securities market reopens at the end of the day to allow households to convert any unused money balances into government bonds. Cochrane describes this model as cashless and frictionless. But as I will explain below, both of these descriptions are misleading. Cochrane (2005, p. 513) later introduces the concept of a completely cashless model and it is that model that is also completely frictionless and the source of the controversy over the FTPL.

**Cochrane’s ‘cashless’ model.**

In the cashless version of the model Cochrane amends Sargent’s specification so that households can return to the securities market at the end of the day and trade any
unwanted cash (money) for more government bonds. In that case expression (6) does not hold in the ‘cashless’ version of the model. Cochrane (2005, p. 508) states that the absence of (6) is the only difference between the two versions of the model.

In Cochrane’s cashless model equilibrium in the goods, money and assets markets are described by the following equations:

\[
\begin{align*}
\beta^j \frac{u'(e_{t+j})}{u'(e_t)} &= m_{t,t+j}, \quad (17) \\
M_t &= 0 \quad (18) \\
\frac{B_{t-1} + M_{t-1}}{P_t} &= \sum_{j=0}^{\infty} E_t(m_{t,t+j} s_{t+j}) \quad (19)
\end{align*}
\]

The behaviour of households in the goods market remains unchanged. The key difference is that in the cashless model households have converted all their money into government bonds overnight. Hence equation (18) and the statement that money demand is zero – from equation (6). The idea here is that if interest rates are positive, \( r_{t+j} > 0 \), then households hold no money overnight – they buy government bonds when the financial markets open at the end of the day. In this state of the model demand for overnight money would only be positive if nominal interest rates were zero.

However, households must still use money to buy goods the next day subject to the CIA constraint on intraday trade. They start the day by selling bonds for currency to the government, using currency to pay taxes and then using the rest of the money so obtained to buy goods and so on. Consequently, I would argue that it is not helpful to describe this model as ‘cashless’. It differs from the monetary version only in the
distribution of money balances ‘overnight’. From equation (19) it is apparent that the
government holds the money ‘overnight’.

The same argument explains why the cashless model is not frictionless - all the
restrictions imposed by Sargent (1987), in the form of the intraday CIA constraint,
still remain. So Clower’s observation that money is a welfare reducing constraint
applies to the cashless version of the model. Consequently, although Cochrane has
removed a ‘little friction’ in the cashless version of the model it is also not helpful to
describe this version of the model as frictionless. The ‘big friction’ generated by the
CIA constraint still applies to intraday trade so the model is still a ‘monetary’
Walrasian general equilibrium model subject to the constraints imposed on
households (the CIA constraint) to generate a non-zero exchange value for fiat money.
To eliminate all frictions \((mrs \neq mrt, \text{ and Sargent’s restrictions on exchanges})\)
Cochrane takes the next step and removes the CIA constraint, the ‘big friction’, to
produce what he calls the completely cashless model. This model is a moneyless
model and is truly frictionless, as all the constraints on households to generate a non-
zero exchange value for money have been removed.

\textit{Cochrane’s completely cashless and frictionless model}

Cochrane (2005, p. 513) offers what he calls the completely cashless and therefore,
frictionless interpretation of his model when he states:

“The cash in advance constraint plays no essential role in the equilibrium. …For any
equilibrium of the frictionless [i.e. ‘cashless’] model stated so far, the same
equilibrium holds if we eliminate the cash constraint and …eliminate intraday
cash.”

In this case money is eliminated altogether, money demand disappears and along with
it any \(M\) in equation (19). The model is reduced to the two equations:
\[ \beta^j \frac{u'(e_{r+j})}{u'(e_j)} = m_{r+j}. \]  

(17)

\[ \frac{B_{r-1}}{p_i} = \sum_{j=0}^{\infty} E_j (m_{r+j} s_{r+j}) \]  

(19')

In equation (19') the absence of \( M_{r-1} \) indicates that the model is completely cashless and hence frictionless in the sense of both Cochrane and Sargent/Clower. It is on the basis of the completely cashless and frictionless version of the model that Cochrane makes his strongest claims about the FTPL and it is in this context that large conceptual hurdles emerge.

III  Interpreting Cochrane’s model.

The first point to note about the monetary and ‘cashless’ versions of Cochrane’s model (which I treat as analytically equivalent for the reasons given above) is that Walras’s Law is an inevitable property of a well-specified Walrasian general equilibrium model so applies to Cochrane’s analysis. Hence, we can follow Patinkin (1965, chapters 3, 10, 11) and use the three equations (12), (14) and (15) to derive market-clearing loci for Cochrane’s monetary version of the model in \((r, p)\) space (and the cashless version for reasons explained above). The market clearing conditions provided by Cochrane (2005, p. 509) are simply:

\[ c_i = e_i \]  

“goods market”  

(12’)

\[ M_i = M_i^s \]  

“money market”  

(14’)

\[ B_i = B_i^s \]  

“bonds market”  

(15’)

For each date and state of nature, expressions (12’) to (15’) can be used to conduct what Patinkin called ‘market experiments’ to derive the market clearing loci for each of the goods, money and bonds markets in \((r, p)\) space. By Walras’s Law only two of
these three market-clearing loci are independent so only two are needed to provide the equilibrium solution \((r, p)\). This analysis is familiar from Patinkin’s *Money Interest and Prices*, chapters 10 and 11. See, in particular, Figure 11-2, Patinkin (1965, p. 259). Which of the market-clearing loci is dropped is entirely arbitrary. Thus, in Chapter 10 Patinkin drops the bond market and, before proceeding to drop the money market in chapter 11, he notes, Patinkin (1965, p. 253):

“We shall now show that the conclusions reached in the preceding chapters by an analysis of the commodity and bond markets can also be reached – in a somewhat more familiar way – by an analysis of the money market. This equivalence is, of course, a simple implication of Walras’s Law”

Thus in a well-specified Walrasian model with fiat money, where money is incorporated either in the utility function or as a cash-in-advance constraint\(^5\), there is no basis in the model for distinguishing between a quantity theory of the price level or a bond (fiscal) theory of the price level or, for that matter, the commodity theory of the price level. If we drop the goods market and leave the money and bonds markets what do we call the theory of the price level in that case? Notice, that by Walras’s Law the money market locus for the cashless version of the model could be dropped irrespective of who is holding the fiat money balances so, at least from the perspective of Walras’s Law, the latter refinement adds nothing of analytical significance.

The situation changes significantly when we consider the completely cashless, frictionless version of the model. In this version of Cochrane’s model, fiat money (cash) has been eliminated altogether, and money is redefined as an income earning asset; hence the title of Cochrane’s paper –money as stock. This avoids Clower’s

---

\(^5\) As Feenstra (1986) explained, under certain conditions the cash-in-advance constraint is simply a special case of the money in the utility function approach. See Patinkin (1989, p. xxxiii).
problem and the fact that fiat money has zero exchange value in Walrasian general equilibrium models (Hahn, 1965, Sargent 1987, p. 136) but raises conceptual hurdles of the sort noted by Buiter and McCallum.

Before we consider those, note that in its completely cashless, frictionless state, Cochrane’s model still satisfies Walras’s Law. But as the model has been reduced to only two market-clearing loci, those that can be derived from expression (12’) and (15’) backed by expressions (17) and (19’), the relevance of Walras’s Law is moot – both market clearing loci are needed to solve for the equilibrium \((r, p)\) solution. The variables \(r\) and \(p\) do, however, require careful re-interpretation (and re-definition) in the completely frictionless model. The quantity of money no longer exists so the quantity theory cannot be proposed as a theory of the price level. What then determines the price level in this version of the model? The simple answer is that the price level, as defined in the monetary and cashless versions of the model, no longer exists. It is replaced by something else. Hence the objection by McCallum (2003, p.634) that the analysis is misleading and Buiter’s (2002) objection that the fiscal theory of the price level is an intellectual bridge too far. They object to the reinterpretation of the concept of the price level offered by Cochrane.

Specifically, Buiter (2004, p. 31, emphasis added) argues that although Cochrane’s model can determine a relative price of two commodities expressed in terms of the numeraire (e.g., phlogiston) it is obviously not possible for the model to determine the price of phlogiston (the numeraire):

“All two commodities priced in phlogiston (or any imaginary and non-existent numeraire) will have a well-determined relative price. Determining the price of phlogiston (the numeraire) when phlogiston does not exist except as a word, is an intellectual bridge too far.”
To clarify the matter, recall Patinkin’s (1965, pp. 15-17) distinction between types of money and prices in the Walrasian general equilibrium model. He distinguished between two types of money in his model. The abstract unit of account for purposes of record keeping but which had no physical existence -we could follow Buiter and call it phlogiston- and fiat money that acts as the physical medium of exchange. Corresponding to the two types of money were two types of prices and Patinkin labelled them accounting prices and money or absolute prices respectively. In addition Patinkin noted that there were real or relative prices that represent the prices of commodities in terms of one another. Finally, Patinkin (1965, p. 403) defined the price level in his model as 

\[ p = \sum_{j=1}^{n} w_j p_j \]

where the \( w_j \) are the prices of goods in terms of the unit of account, the \( w_j \) are known weights and the \( n^{th} \) good is treated as paper money.

Applying these definitions to Cochrane’s model reveals the conceptual hurdles to which Buiter and others object. The frictionless state of Cochrane’s model is one in which fiat money has been eliminated so we are back in Sargent’s (1987, section 3.3) statement of a Lucas model of asset prices. Consequently, all assets, shares in trees and government bonds must be paid for with fruit even if their prices are quoted in $s or units of phlogiston.

In terms of the abstract unit of account, phlogiston, the prices of the fruit and the bond and stock can be written as \((p_f, p_b, p_s)\). If bonds have taken over the role of money, as Cochrane (2005, p. 506) suggests when he claims that the completely cashless
economy can work just as well as a monetary economy if government bonds act as the medium of exchange, the respective relative prices are simply \( p_j / p_b \cdot 1, p_j / p_b \) where the \( p_j; j = s, b, c \) are the accounting prices. Patinkin’s definition of the price level in terms of any numeraire prices then reduces to \( p = \sum_{j \neq b} w_j p_j \cdot (j \neq b) \). So in a simple technical sense the price level in Cochrane’s model can be defined in terms of numeraire prices, even units of phlogiston. Why can’t this be interpreted as a proxy for the price level as defined in the monetary version of the model? The answer is that the ‘price level’ defined in terms of accounting, or numeraire, prices is arbitrary or indeterminate. It has no theoretical significance. As Patinkin (1965, p. 16) explains:

“The accounting price of a given good is distinctive in having no operational significance for the market….. the statement that “the accounting price of X is 4” – in the absence of additional information on the accounting price of at least one other good –gives us no idea what we must do to acquire a unit of good X.”

A similar argument applies to the definition of the price level in the completely cashless, frictionless version of Cochrane’s model. The arbitrary nature of the numeraire or accounting prices, they can take on any absolute values so long as they reflect the relative commodity exchange ratios generated by the model, is transferred directly to the definition of the price level, it is entirely arbitrary and has no analytical or theoretical significance. In this sense Cochrane fails in his attempt to demonstrate that the FTPL is a theoretically coherent possibility in the context of a well-specified Walrasian general equilibrium model. In the completely cashless and frictionless model the FTPL is reduced to an arbitrary definition of the ‘price level’ in terms of accounting prices. This is one element of the substance to the objection by Buiter and McCallum.

6 Buiter (1998, p. 29) makes this quite clear when he argues: “In the demonetised economy, money has become solely the numeraire. We know from general equilibrium theory that the numeraire need not be a good or bundle of goods in the commodity space. Indeed the numeraire could be something entirely fictitious, such as phlogiston. Only relative prices matter and are determinate.”
What the model does determine is the relative price of the consumption commodity, fruit, in terms of bonds \((p_c/p_b)\) or the relative price of shares in terms of bonds \((p_s/p_b)\). And, although these prices are expressed in terms of units of phlogiston, they represent quantities of fruit so the model is essentially one of barter (actually perfect barter). Cochrane’s completely cashless and frictionless model is obviously a model of asset prices. It determines the price of an asset in terms of the consumption good. Redefining the asset as money then enables Cochrane to define the price level as the relative price of consumption goods and asset-money. To see this, consider Cochrane’s formal derivation of what he calls the price level in his frictionless model. Cochrane (2005, p. 511) draws out these features of his model when he observes that, with a utility function of the form, \(u(c) = c^{1-\gamma}\) and \(e_t = e\), \(B_t = B\), \(M_t^s = 0\) and \(s_t = s\), all positive and constant over time, equations (17) and (19’) simplify to:

\[
\begin{align*}
m_{t,t+1} &= \beta \\
p_t &= p = (1 - \beta) \frac{B}{s}
\end{align*}
\]

The discount factor is constant, *nominal interest rates are said to be positive* and the *price level is said to be positive and constant*. So if true, this would amount to the determination of the price level in the perfectly cashless and frictionless model.

However, a moment’s reflection will reveal that this argument fails at several points. First, the variable \(p_t\) in equation (19’’) obviously does not correspond to the price level, \(p_t\) as defined in equation (14). Equation (19’’) is simply a special case of the
asset pricing formula of the type discussed by Sargent (1987, p. 96) so a new variable, \( q_t \), the consumption good value of a share, should be introduced here. That is:

\[
q_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{u'(d_{t+j})}{u'(d_t)} d_{t+j}.
\]  

(20')

Where \( q_t \) is defined as the real asset price (measured in terms of the consumption good, fruit), not the price level, \( p_t \) from equation (14), and \( d_t \) are the dividends, the fruit from the tree, paid to holders of titles to these assets, all measured in terms of the consumption good. For the simple utility function \( u(c) = \ln c \), Sargent’s asset pricing formula becomes:

\[
q_t = E_t \sum_{j=1}^{\infty} \beta^j d_j \quad \text{or} \quad q_t = \frac{-\beta}{1-\beta} d_t.
\]  

(21')

Expression (19'') is simply another special case of this form of solution represented by (21'). The Lucas tree model without a CIA constraint determines the real price (measured in terms of the consumption good) of the security backed by the tree. Thus Cochrane’s completely cashless and frictionless model determines the ‘price’ of the stocks (shares in trees), \( q_t \), as measured in terms of consumption goods at period \( t \) per share held during period \( t \) and where the dividends are also paid in period \( t \) consumption good. Consequently, the LHS variable in equation (19’’) should be \( q_t \), the consumption good value of shares in the trees and is clearly not what is traditionally understood by the price level\(^7\).

---

\(^7\) Hoover (1988) pointed to a similar conceptual hurdle in Fama’s (1980) attempt to apply Walrasian general equilibrium theory to monetary economics.
Second, the mathematical identity between stock and bonds occurs because bonds and stock are effectively perfect substitutes (trade one for one) in the sense that arbitrage maintains equality between their prices. Hence the relationship between expressions (1) and (2) in the frictionless model is not by analogy but an inevitable property of the model. Yields on government bonds, in terms of fruit, must be the same as on shares in trees. Pricing shares in terms of fruit simultaneously prices government bonds in terms of fruit. We could just as easily use the model to determine the stock theory of the ‘price level’ rather than the FTPL. Interpreting stock (an income earning asset) as money is then the basis for Cochrane’s claim that as the model determines the consumption good value of asset-money it is a theory of the price level. This logic fails because the price level and the consumption good value of an asset are conceptually and analytically two quite different things.

Third, equation (19’’) contains $B$ which was initially defined as the face value of a nominal government bond that was title to a flow of interest paid in money. But in the completely cashless, frictionless model there is no fiat money in which $B$ can be paid. Buiter (1999) makes this point clear. Consequently there are no nominal interest rates; how much money must be given up today in exchange for a unit of money tomorrow. What interest rates there are, are commodity rates, how much fruit must be given up today in exchange for a unit of fruit tomorrow. Expressing this intertemporal exchange ratio in terms of numeraire prices, e.g., phlogiston, does not convert it into a monetary or nominal rate.

Finally, the frictionless model has no role for money as a medium of exchange - be it fiat money or asset money. This follows from the time-0 form of the auction that
underlies the frictionless model and it is this form of auction that effectively renders
the model frictionless (see Ljungqvist and Sargent (2004) for a discussion of the
properties of this form of auction.) As a model of efficient or perfect barter there is
simply no need for any transactions technology –the model does not specify any and
none is required (Rogers 2005). Cochrane’s suggestion that asset-money can become
the medium of exchange is therefore inconsistent with this property of the completely
cashless and frictionless model. Assets cannot be imbued with the medium of
exchange function of money in this version of the model without re-introducing
frictions.

Consequently, Buiter, McCallum and others are right to reject Cochrane’s concept of
the price level, not on semantic grounds, but on conceptual and theoretical grounds.
The concept of money changes as we move through the various versions of
Cochrane’s model as does the concept of the price level. The concepts are not
comparable across versions of the model. Furthermore, the numeraire prices of assets
and fruit are entirely arbitrary as is any ‘price level’ defined in terms of them.
Cochrane’s completely cashless and frictionless model determines the consumption
good value of assets and relative prices of assets and the consumption good all
measured in units of fruit. It does not determine the price level as defined in monetary
Walrasian general equilibrium models or as measured by the GDP deflator or CPI.

IV Concluding remarks

In this paper I have argued that neither the cashless nor the completely cashless and
frictionless versions of the model presented by Cochrane can provide a theoretically
coherent framework for the FTPL.
Cochrane’s formal *cashless model* is not completely cashless or frictionless. It is a monetary model of the Patinkin type in which money is a welfare reducing friction. The formal cashless model, equations (17) to (19), does not become completely cashless or frictionless when money or cash is converted to bonds overnight. It is not completely cashless because ownership of cash has simply been transferred to the government and the CIA constraint (a big constraint) still applies to intraday trade. It is not frictionless because the CIA constraint imposes the medium of exchange function of money as a welfare reducing friction on intraday trade.

In that respect Cochrane’s *formal* ‘cashless’ version of the model has all the properties of Patinkin’s model in *Money, Interest and Prices*. Cochrane’s model in both its cashless and monetary versions is a monetary model of sorts but one in which money has been imposed when it is not needed. Hence in a well-specified Walrasian general equilibrium model fiat money is a ‘big friction’. The model has the property that the price level, as the inverse of the purchasing power of the money stock, can be determined but there is no basis in the model to claim precedence for the bond market over the money market, or indeed the goods market. The FTPL cannot take precedence over the QTPL in Cochrane’s ‘cashless’ model – equation (19) has no precedence over equation (14).

Cochrane’s informal discussion of the model relies more substantially on the *completely cashless and frictionless* version. But that version cannot provide a foundation for any *theory* of the price level, let alone the FTPL. Contrary to Cochrane’s claim, the FTPL is not a coherent theoretical possibility in a completely cashless, frictionless and well-specified Walrasian equilibrium system. The
completely cashless, frictionless version of a well-specified Walrasian equilibrium system, one that completely eliminates the ‘big friction’ at the short end of the yield curve, effectively rules out any role for the concept of a price level employed in the monetary version of the model. A numeraire price level can be defined in terms of a set of numeraire prices for assets and commodities but that price level is itself arbitrary because it is defined in terms of the arbitrary numeraire, as Patinkin explained. Thus no version of Cochrane’s well specified Walrasian general equilibrium model, monetary, cashless or completely cashless and frictionless, offers a coherent theoretical foundation for the FTPL.

Cochrane’s stock-bond analogy also raises conceptual hurdles when subject to scrutiny. Removing the big friction represented by the CIA constraint reduces Cochrane’s model to one equivalent to the Lucas tree model without a CIA constraint. That is essentially a model of asset pricing and not a model of the price level. The model can determine the consumption good value of shares in trees and by arbitrage the consumption good value of government bonds. Interpreting the consumption good value of a government bond as the price level requires that we (a) interpret the price level as a relative price and (b) interpret the bond as money so we have the consumption good price of money as the concept of the price level. In addition, the frictionless version of a well-specified Walrasian general equilibrium model has no role for asset money, any more than it does for fiat money, as a medium of exchange.

In this paper I have also pointed out the source of the conceptual hurdles generated by Cochrane’s use of the completely cashless and frictionless model. The latter is an inherent property of the time-0 auction that underpins the Walrasian general
equilibrium system employed by Cochrane—such a model is best interpreted as a model of perfect barter. Attempts to employ the Walrasian system as a basis for monetary theory have thrown up similar conceptual hurdles in the past, as with the MIU of Patinkin and CIA constraint of Clower. Failure to recognise these properties of the various states of his model leads to the conceptual hurdles acknowledged by Cochrane. Thus, despite the distraction of the debate about Walras’s Law there are grounds for concluding that the application of the Walrasian general equilibrium model as a basis for the FTPL is logically and conceptually flawed—as Buiter (1999, p.1) claimed—and not relevant to actual economies, as McCallum (2003, p. 634) suggested.
References


