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Paul Pezanis-Christou

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Paul Pezanis-Christou*

School of Economics, The University of Adelaide

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Abstract

The paper reports on the effects of one-sided imperfect information on bidding behaviour in simultaneous and sequential first-price auctions of non-identical objects when bidders have multi-unit demands. The analysis provides the following four main results. First, when different objects are to be sold in sequence, the seller maximises her expected revenues by selling the most valuable object first. Second, the more the objects are different and the more the sequential format favours the informed bidder. Third, by switching the order of sales, the seller may want to change her initial preference for a simultaneous format (in which bidders submit object-specific bids) to one for a sequential format. Fourth, sequential auctions are mostly preferred by the seller when the objects are likely to be of low value and the precision of the informed bidder's signal is low.

Keywords: multiple-object auctions, sequential and simultaneous procedures, first-price auctions, asymmetric bidders, multi-unit demands, common value, price trends, order of sales.

J.E.L. Classification: C7, D4, D44, D8.

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Besides which pricing-rule to use, a question that pervades the auctioning of multiple objects is whether to auction them 'all in one go' (simultaneously) or 'one after the other' (sequentially). While much of the auction literature studied and compared the equilibrium properties of various pricing-rules, surprisingly little is known about the simultaneous *versus* sequential use of a particular pricing-rule, especially when the objects are non-identical.

Milgrom and Weber (2000) provide a comprehensive comparison of various auction procedures for identical objects and show that the sequential versions of the first- and second-price auctions are revenue superior to their simultaneous versions (i.e., discriminatory and uniform-price auctions, respectively) when bidders are symmetric and have affiliated signals. The intuition for this prediction is that the revelation of earlier winning bids reduces the potential for a winner's curse in later stages and induces a more aggressive bidding. This, in turn, leads to higher expected revenues than single-stage auctions where no information is released during the auction. However, subsequent work has indicated that this ranking is sensitive to the model's assumptions, and in particular to whether bidders have unit-demands or are symmetrically informed.

Hausch (1986) shows that in the case of first-price auctions, the revenue superiority of the sequential format does nomore necessarily hold when the assumption of unit-demands is relaxed. This is because assuming multi-unit demands triggers an incentive for bidders to bid low in the first stage so as to deceive their competitor into thinking they received a low signal and win the second-stage object with a profit (the so-called *deception* effect). Actually, depending on the quality of the signals received, a bidder's equilibrium strategy can either be to conceal her high signal in the first stage (in which case the seller prefers the sequential format) or to randomise between concealing and revealing it (in which case the seller prefers the simultaneous format). By concealing her high signal, the first stage of the sequential auction delivers no information for the second stage so the potential for the winner's curse, which calls for a cautious bidding, exists only for the second object. In simultaneous (single-stage) auctions, however, the potential exists for both objects and depresses the seller's revenues. When bidders randomise, the fear of a winner's curse spreads over both units which, together with the *deception* effect, yields the opposite revenue ranking. Such a setting leads the seller to prefer the sequential format when the objects are likely to be of high value and the bidders receive signals of low precision.[1] Yet, the assumption

[1] Clearly, this result requires that winning bids are disclosed at the end of the first stage. Mezetti, Pekeč and Tsetlin (2008) tackled this assumption in sequential auctions with affiliated values and unit-demands. They consider uniform-price auctions and analyse the cases where the objects are sold in a sequence of two (uniform-price) auctions or in a single stage. They show in particular that the seller always prefers the latter when the first-stage winning bids are not disclosed; otherwise the revenue ranking depends on the auction's

of symmetrically informed bidders is central to these predictions. Engelbrecht-Wiggans and Weber (1983) relax this assumption by considering the polar case of a perfectly informed bidder competing against an uninformed one. Their study puts in perspective the dilemma faced by the informed bidder: by submitting a "serious" bid (i.e., suggesting that the objects are of high value) in any non-final stage of a sequential auction, she would reveal her good signal and make a zero profit in the remaining stages. In the unique equilibrium of this game, the informed bidder randomises the submission of "serious" bids over all stages, thereby letting the uninformed bidder realise positive expected profits in the stages preceeding the submission of her "serious" bid. Interestingly, when there are only two objects for sale, such randomisation leads the seller to prefer the sequential format when the objects are likely to be of low value; which is opposite to Hausch (1986). Hörner and Jamison (2008) study the infinitely repeated version of this game and show that if one of the bidders is informed and discounts the future enough, then she will reveal her information in finite time; thereby making the sequential format most profitable for the seller.[2]

As for the simultaneous *versus* sequential auctioning of non-identical objects, the only predictions available pertain to second-price auctions of complementary assets. When information is incomplete and bidders are not budget constrained, Krishna and Rosenthal (1996) numerically show that the simultaneous (sequential) format is revenue superior for low (high) levels of asset complementarity. In the presence of budget constraints and complete information, Benoît and Krishna (2001) show that the sequential format is then no inferior, and sometimes superior, to the simultaneous one if the objects are sufficiently complementary. Goeree, Offerman and Schram (2006) provide experimental evidence on the properties of various first- and ascending-price auctions for non-identical assets (when buyers have unit-demands). Their 'model-free' study indicates that when bidders have affiliated signals, selling the assets sequentially by first-price auctions yields the highest revenues to the seller, no matter if bidders are symmetrically or asymmetrically informed.

The sequential sale of non-identical objects also raises the question of which object to sell first? This question has mostly been examined in the context of second-price sealed bid auctions and recommendations for the seller are to sell first the object for which the bidders' private values are most dispersed (Bernhardt and Scoones, 1994) or the one that bidders value most (Benoît and Krishna, 2001, and Chakraborty, Gupta and Harbaugh,

specifics in a nontrivial way.

[2] The disclosure of winning bids is again essential for these predictions to hold. Virág (2007) studies a repeated first-price common value auction where only the winner's identity is disclosed, not the submitted bids. This embeds the auction game in one of information acquisition which increases the seller's expected revenue when compared to the simultaneous case.

2006).[3] Pitchik (2009) discusses how the outcomes of a sequential auction relate to the buyers' budget constraints and valuations, the order of sales and the price-rule used. In particular, she provides conditions for the seller's revenue to only depend on the order of sales and for which the expected price of an object increases the later (the earlier) it is sold in a sequence of second-price (first-price) auctions. Eventually, the experimental finding of Goeree *et al.* (2006) on the superiority of sequential first-price auction holds only when the assets are sold by decreasing order of quality.

This paper considers a model *à la* Engelbrecht-Wiggans and Weber (1983) to study the joint effects of one-sided incomplete information and assets' heterogeneity on the formation of prices in simultaneous and sequential first-price auctions. When the assets are indivisible, it is shown that the equilibrium strategy for the informed bidder in the first stage consists either in *(i)* concealing her signal, *(ii)* randomising between concealing and revealing it, or *(iii)* revealing it. In the latter case, which occurs when the first object is of sufficiently *higher* value than the second, the seller is best off using the sequential format. In case *(ii)*, the seller should decide between the simultaneous and the sequential formats only after considering selling the most valuable object first in a sequential sale. Overall, sequential auctions become more profitable as the objects are likely to be of low value and the informed bidder's signal is sufficiently precise. In case *(i)*, which occurs when the first object is of sufficiently *lower* value than the second, the seller prefers a simultaneous format but would always be better off selling the most valuable object first in a sequential sale. When the asset is perfectly divisible, then the seller prefers to sell a fraction of the asset in the first stage so as to induce the informed bidder to reveal her signal and extract all the bidder's surplus in the second stage. Finally, with reference to the literature on the declining price anomaly (see Ashenfelter, 1989, and Sosnick, 1963, for an early account of this phenomenon), this study provides an additional explanation for the occurrence of non-constant price trends and shows that the Law-of-One-Price may hold in sequential first-price auctions even if the objects are different and bidders are asymmetric.

The following section determines the equilibrium bidding strategies for simultaneous and sequential auctions. Section 2 reports on the possible price trends in sequential auctions, Section 3 reports on the seller's revenues and Section 4 concludes.

[3] Elmaghraby (2003) studies sequential second-price procurement auctions with capacity-constrained bidders and discusses conditions on the suppliers' cost functions to be satisfied for the allocation of assets (jobs) to be efficient.

1 Equilibrium behaviour

Assume two risk neutral bidders who compete for the purchase of an asset of unknown value $V = \{\underline{V}, \overline{V}\}$ which is distributed according to some common knowledge distribution. This asset can be one of the following two forms. It can either consist of two indivisible objects, the first being of value $V_1 = \sigma V$ with $0 < \sigma < 1$ and the second of value $V_2 = (1 - \sigma)V$, or it can be perfectly divisible in two parts. Each bidder has a demand for the whole asset and one of them has some information about its value V. Henceforth, it is assumed that bidder 1 receives a private independent signal S about V which is either "Low" or "High" so that $S = \{L, H\}$ whereas bidder 2 is uninformed and receives no signal. Bidder 1's signal is imperfect and $p(S = H|V = \overline{V}) = p(S = L|V = \underline{V}) = p \in [0.5, 1]$ so she has proprietary information in the sense of Milgrom and Weber (1982). Let $p(s)$ denote the probability that bidder 1 receives signal s and $v(s) = E[V \mid S = s]$, so that $v(H) > v(L)$.

In the case of two indivisible objects, the seller has the option to sell them either simultaneously or sequentially by means of two first-price auctions (i.e., one for each object). Otherwise, she has to decide what fraction of V to sell first; the remainder being sold in a second auction. Whenever a sequential format is used, all submitted bids are revealed at the end of the first auction.

1.1 Simultaneous auctions

To determine the equilibrium strategies for this format, it is enough to consider the case where there is only one object of common value V for sale and to observe that since the objects display no synergy to bidders, the multi-object case simply is a repetition of the single object case, modulo σ. For expositional convenience, the objects are identified as the "first " and "second" object eventhough there is no reference to the timing of sales. It is easily shown that the equilibrium strategies for this format must satisfy the following properties:

- No bidder bids less than an object's expected value for a Low-signalled bidder 1, i.e., $\sigma v(L)$ for the "first" object and $(1 - \sigma)v(L)$ for the "second".[4]
- Bidder 2 bids no more than bidder 1's *ex ante* expected values for the objects, i.e., $\overline{b}^1 = \sigma \{p(L)v(L) + p(H)v(H)\}$ for the "first" object, and $\overline{b}^2 = (1-\sigma) \{p(L)v(L) + p(H)v(H)\}$

[4]If that was not the case, then bidders would realise a positive expected profit for each object they win which, given bidders' competition, cannot constitute an equilibrium.

for the "second".[5] As a result, bidder 1 does not bid more than $\bar{b}^1$ for the "first" object, nor more than $\bar{b}^2$ for the "second" object.

- Bidder 2's equilibrium strategy must be *mixed* of the range $[\sigma v(L), \bar{b}^1]$ for the "first" object and over the range $[(1-\sigma)v(L), \bar{b}^2]$ for the "second".[6]

What immediately follows considers the "first" object; the "second" objects is treated in a similar way and the equilibrium expressions are reported in the Appendix. As a Low-signalled bidder 1 bids $\sigma v(L)$, her expected payoff equals 0. Upon receiving a High signal, she wins the auction if her bid b is greater than the one of bidder 2 who draws her bid from a probability distribution F_2^1. Therefore, bidder 1's expected payoff from winning the first object is $[\sigma v(H) - b]F_2^1(b)$. Taking its first order condition with respect to b and solving it for f_2^1, the density of F_2^1, yields the differential equation $f_2^1(b) = F_2^1(b)/[\sigma v(H) - b]$ which has for boundary condition $F_2^1(\bar{b}^1) = 1$, and for solution

$$F_2^1(b) = \frac{\sigma[v(H) - v(L)]p(L)}{\sigma v(H) - b} \text{ with } b \in [\sigma v(L), \bar{b}^1]. \tag{1}$$

As bidder 2 receives no signal, her expected payoff from winning the first object is $[\sigma v(H) - b]p(H)F_{1H}^1(b) + [\sigma v(L) - b]p(L)$, where F_{1H}^1 stands for the bid distribution of a High-signalled bidder 1. The first (second) part of this expression stands for the expected payoff from out-bidding a High-signalled (Low-signalled) bidder 1. Differentiating with respect to b and solving the first order condition for f_{1H}^1, the density of F_{1H}^1, yields $f_{1H}^1(b) = (p(L) + p(H)F_{1H}^1(b))/\{[\sigma v(H) - b]p(H)\}$ which has for boundary condition $F_{1H}^1(\bar{b}^1) = 1$, and for solution

$$F_{1H}^1(b) = \frac{[b - \sigma v(L)]p(L)}{[\sigma v(H) - b]p(H)} \text{ with } b \in (\sigma v(L), \bar{b}^1]. \tag{2}$$

It is shown in the Appendix that these bid distributions, together with a bid of $\sigma v(L)$ for a Low-signalled bidder 1, constitute a Nash equilibrium. Substituting these distributions in the bidders' expected payoff functions for each object leads to the following expressions for the bidders' total *ex ante* expected payoffs

[5] If that was the case, the uninformed bidder 2 would realise a negative *ex ante* expected profit for each object she wins which is ruled out in equilibrium.

[6] If that was not the case, say bidder 2 always bids some bid $b_1^\circ \in [\sigma v(L), \bar{b}_1]$ for the first object and $b_2^\circ \in [(1-\sigma)v(L), \bar{b}_2]$ for the second, then bidder 1 would always be better off bidding just below (above) b_1° and b_2° upon receiving a Low (High) signal. In this case, bidder 2 would always win (lose) the objects when the bidder 1 receives a Low (High) signal which cannot be sustained as an equilibrium.

$$p(H)p(L)[v(H) - v(L)] \text{ for bidder 1,} \qquad \text{and} \quad 0 \text{ for bidder 2.} \tag{3}$$

Figure 1 displays two examples of equilibrium bid distributions when the objects' common value V is equal to $\overline{V} = 10$ with probability $\theta = 0.8$ and $\underline{V} = 0$ otherwise, $\sigma = 0.5$ or 0.6, and $p = 0.9$.

< *Figure 1 about here* >

1.2 Sequential auctions

The analysis of the sequential format involves two possible situations; one in which the objects are indivisible, i.e., with a fixed $\sigma \in (0, 1)$, and one in which the object is perfectly divisible and the seller chooses the fraction $\tau \in (0, 1)$ of the asset to be sold in the first auction stage.

1.2.1 Indivisible objects

When the objects are to be sold in sequence, there exist a multiplicity of equilibria which can be reduced to a single one if a Low-signalled bidder 1 never bids more than $\sigma v(L)$. As highlighted by Engelbrecht-Wiggans and Weber (1983), a High-signalled bidder 1 then faces the dilemma of when to reveal her signal. If she does so in the first stage, then she would earn zero expected profits in the second whereas if she conceals her High signal in the first stage, then she postpones competition to the second stage. In the unique equilibrium of this game, a High-signalled bidder 1 must therefore randomise the revelation of her signal in the first stage. To capture bidder 1's dilemma, it is assumed that in the first stage she bids $a = \sigma v(L)$ upon receiving a Low signal; and that she draws a bid from G_{1H} defined on $[\sigma v(L), \beta_1^1]$ with $G_{1H}(\sigma v(L)) > 0$ upon receiving a High signal. That is, bidder 1 reveals her High signal with probability $1 - G_{1H}(\sigma v(L))$ in the first auction. As for bidder 2, it is again easily shown that her equilibrium strategy for the first object must be *mixed* over some range $[\sigma v(L), \beta_2^1]$, and that since bidder 1 will never bid more than β_2^1 for the first object, one must have a common maximum possible bid for the first object: $\beta_1^1 = \beta_2^1 = \beta^1$. Finally, the determination of the equilibrium requires that (the uninformed) bidder 2 always wins ties against bidder 1, as in Hörner and Jamison (2008) (or that bidder 2 never bids $\sigma v(L)$, as in Engelbrecht-Wiggans and Weber, 1983).

To determine the bidders' equilibrium strategies, note that in the presence of non-identical objects, bidder 1 may have an incentive to always reveal or to always conceal her signal in the first stage. It is shown in the Appendix that if the first object is of a sufficiently *higher* value than the second, the bidders' equilibrium behaviour consists in bidding as for the "first" object in a simultaneous sale and to bid the second object's expected value, $v(L)$ or $v(H)$ (depending on bidder 1's first stage bid) in the second auction. This happens when $\sigma > \frac{1}{p(L)+1}$. On the other hand, if the first object is of a sufficiently *lower* value than the second, both bidders are best off bidding $\sigma v(L)$ in the first auction to conceal their High signals, and to bid for the second object as they would for the "second" object in a simulatenous auction; which happens when $\sigma < \frac{p(L)}{p(L)+1}$.

What follows determines the equilibrium predictions for the case where bidder 1 randomises between the two alternatives in the first stage, i.e., $\frac{p(L)}{p(L)+1} < \sigma < \frac{1}{p(L)+1}$. Starting from the second stage, the bidders' equilibrium strategies will depend on bidder 1's first stage bid. There are two cases to consider.

Bidder 1 bids $a > \sigma v(L)$ in the first stage. She thus reveals her High signal and earns a zero profit in the second stage because both bidders then bid the second object's expected value given that bidder 1 is High-signalled, $V_2 = (1-\sigma)v(H)$.

Bidder 1 bids $a = \sigma v(L)$ in the first stage. As she does not reveal her signal, bidder 2 revises her prior about bidder 1 being Low-signalled given that she bid $a = \sigma v(L)$ and, consequently, the *ex ante* expected value of the second object for bidder 1 which is bidder 1's maximum possible bid for this object. That is,

$$p(L|a = \sigma v(L)) = \frac{p(L)}{p(L) + p(H)G_{1H}(\sigma v(L))} > p(L)$$

$$\text{and} \qquad \tilde{\beta}^2 = \frac{(1-\sigma)\left[p(L)v(L) + p(H)G_{1H}(\sigma v(L))v(H)\right]}{p(L) + p(H)G_{1H}(\sigma v(L))}.$$

Her expected profit for the second stage is therefore equal to

$$[(1-\sigma)v(H) - b]J_{1H}(b)[1 - p(L|a = \sigma v(L))] + [(1-\sigma)v(L) - b]p(L|a = \sigma v(L)),$$

where $J_{1H}(b)$ stands for the second stage bid distribution of the High-signalled bidder 1, and where the first (second) part stands for the expected payoff from outbidding a High-signalled (Low-signalled) bidder 1. Taking the first order condition of this expression with respect to b and solving it yields $j_{1H}(b) = \{p_1(L) + p_1(H)G_{1H}(\sigma v(L))J_{1H}(b)\} / [(1-\sigma)v(H) - b]p(H)G_{1H}(\sigma v(L))$, where $j_{1H}(b)$ stands for the density of J_{1H}. Given the boundary condition $J_{1H}(\tilde{\beta}^2) = 1$, the solution to this differential equation gives the following distribution for bidder 1's bids

$$J_{1H}(b) = \frac{[b - (1-\sigma)v(L)]p(L)}{[(1-\sigma)v(H) - b]p(H)G_{1H}(\sigma v(L))} \text{ with } b \in ((1-\sigma)v(L); \tilde{\beta}^2] \tag{4}$$

As for bidder 1, her expected profit for the second stage is equal to 0 if she is Low-signalled and to $[(1-\sigma)v(H) - b]J_2(b)$ if she is High-signalled; where $J_2(b)$ stands for bidder 2's second stage bid distribution. Maximising this with respect to b and solving its first order condition for $j_2(b)$, the density of J_2, yields $j_2(b) = J_2(b)/[(1-\sigma)v(H) - b]$. The solution to this differential equation, given the boundary condition $J_2(\tilde{\beta}^2) = 1$, yields bidder 2's distribution for the second stage

$$J_2(b) = \frac{(1-\sigma)[v(H) - v(L)]p(L)}{[(1-\sigma)v(H) - b][p(L) + p(H)G_{1H}(\sigma v(L))]} \text{ with } b \in [(1-\sigma)v(L); \tilde{\beta}^2]. \tag{5}$$

Substituting (5) and (4) in the bidders' expected profit functions leads to the following expressions for their second-stage *ex ante* expected payoffs

$$\begin{aligned} &\frac{p(H)(1-\sigma)[v(H) - v(L)]p(L)}{p(L) + p(H)G_{1H}(\sigma v(L))} \text{ for bidder 1,} \\ &\text{and} \quad 0 \text{ for bidder 2.} \end{aligned} \tag{6}$$

Back to the first stage. Consider first bidder 1. If she is Low-signalled, she can do no better than to bid $a = \sigma v(L)$ in the first auction, which earns her a zero expected payoff. If she is High-signalled, then in equilibrium she must be indifferent between bidding $a \in (\sigma v(L), \beta]$ and $a = \sigma v(L)$. In the former case, she would reveal her signal so her expected profit over the two auctions is

$$[\sigma v(H) - a]G_2(a). \tag{7}$$

In the latter case, she would not reveal her High-signal, lose the first auction and earn her second-stage expected payoff which is given by (6). Therefore, $G_2(a)$ must be such that (7) equals her expected payoff in the second stage, that is

$$G_2(a) = \frac{(1-\sigma)[v(H)-v(L)]p(L)}{[\sigma v(H)-a]\,[p(L)+p(H)G_{1H}(\sigma v(L))]} \text{ with } a \in [\sigma v(L); \beta^1], \tag{8}$$

where $\beta^1 = \sigma v(H) - \dfrac{(1-\sigma)[v(H)-v(L)]p(L)}{p(L)+p(H)G_{1H}(\sigma v(L))}$.

Now consider bidder 2. As she receives no signal and earns zero expected profit in the second stage, her total expected payoff is

$$[\sigma v(H) - a]p(H)G_{1H}(a) + [\sigma v(L) - a]p(L), \tag{9}$$

the first (second) part of which stands for the expected payoff from outbidding a High-signalled (Low-signalled) bidder 1. Taking its first order condition with respect to a and solving it for $g_{1H}(a)$ yields $g_{1H}(a) = [p(L) + p(H)G_{1H}(a)] / [\sigma v(H) - a]p(H)$. Since a High-signalled bidder 1 will never bid $a > \beta$ in the first stage, one must also have $G_{1H}(\beta^1) = 1$. Solving this differential equation given the boundary condition $G_{1H}(\beta^1) = 1$ yields

$$G_{1H}(a) = \left(\frac{(1-\sigma)[v(H)-v(L)]}{[\sigma v(H)-a][p(L)+p(H)G_{1H}(\sigma v(L))]} - 1\right)\frac{p(L)}{p(H)} \text{ with } a \in [\sigma v(L); \beta^1]. \tag{10}$$

Setting $a = \sigma v(L)$, and solving for $G_{1H}(\sigma v(L))$ yields

$$G_{1H}(\sigma v(L)) = \frac{\sqrt{(1-\sigma)\sigma p(L)} - \sigma p(L)}{\sigma p(H)}.$$

Substituting this in (4), (5), (8) and (10) gives the following expressions for bidders' equilibrium distributions of bids

$$\begin{aligned} G_{1H}(a) &= \frac{[v(H)-v(L)]\sqrt{(1-\sigma)\sigma p(L)}}{[\sigma v(H)-a]p(H)} - \frac{p(L)}{p(H)} \text{ with } a \in [\sigma v(L), \beta^1], \\ G_2(a) &= \frac{[v(H)-v(L)]\sqrt{(1-\sigma)\sigma p(L)}}{\sigma v(H)-a} \text{ with } a \in [\sigma v(L), \beta^1], \\ &\text{where } \beta^1 = \sigma v(H) - \sqrt{(1-\sigma)\sigma p(L)}\,[v(H)-v(L)], \end{aligned} \tag{11}$$

and

$$J_{1H}(b) = \frac{[b-(1-\sigma)v(L)]\sigma p(L)}{[(1-\sigma)v(H)-b][\sqrt{(1-\sigma)\sigma p(L)}-\sigma p(L)]} \text{ with } a \in [(1-\sigma)v(L), \tilde{\beta}^2],$$

$$J_2(b) = \frac{[v(H)-v(L)]\sqrt{(1-\sigma)\sigma p(L)}}{(1-\sigma)v(H)-b} \text{ with } a \in [(1-\sigma)v(L), \tilde{\beta}^2], \tag{12}$$

where $\tilde{\beta}^2 = 1-\sigma-\sqrt{(1-\sigma)\sigma p(L)}\,[v(H)-v(L)]$,

Notice that, as expected, for $\sigma = p(L)/\left[1+p(L)\right]$, $G_{1H}(\sigma v(L)) = G_2(\sigma v(L)) = 1$, i.e., bidder 1 always conceals her signal and bidder 2 always bids $\sigma v(L)$, whereas for $\sigma = 1/\left[1+p(L)\right]$, $G_{1H}(\sigma v(L)) = 0$, i.e., bidder 1 always reveals her signal, and $G_2(\sigma v(L)) = p(L)^2/\left[1+p(L)\right]$. Substituting (11) in (7) and (9) leads to the following expression for the bidders' total *ex ante* expected payoffs

$$\begin{aligned} &p(H)[v(H)-v(L)]\sqrt{(1-\sigma)\sigma p(L)} \text{ for bidder 1 (in the first or second stage),} \\ \text{and} \quad &[v(H)-v(L)]\left[\sqrt{(1-\sigma)\sigma p(L)}-\sigma p(L)\right] \text{ for bidder 2 (in the first stage).} \end{aligned} \tag{13}$$

It is worth noting here that the difference between bidder 1 and bidder 2's *ex ante* expected payoffs is positive for $\sigma > \frac{p(L)}{p(L)+1}$ and increasing in σ so that bidder 1 benefits more from her private information as the first object is more valuable than the second.

Figure 2 displays the equilibrium bid distributions when the objects' common value V is equal to $\overline{V} = 10$ with probability $\theta = 0.8$ or $\underline{V} = 0$ otherwise, $p = 0.9$, with $\sigma = 0.5$ or 0.6.

< Figure 2 about here >

1.2.2 Divisible object

When the object is divisible in two parts, the seller chooses how much to sell in the first stage; the remainder being sold in the second stage. It immediately follows from the above analysis that the seller maximises her expected revenues when bidder 1 reveals her information in the first stage. This can be achieved by putting up for sale the smallest possible share τ^* such that a High-signalled bidder 1 reveals her signal (i.e., as if she was bidding for the "first" object in a simultaneous auction); that is $\tau^* = \frac{1}{1+p(L)}$. By doing so, bidding for the remainder of the asset in the second auction yields zero expected profits to the bidders. Therefore, the equilibrium bidding strategies are given by (2) and (1), and their total *ex ante* expected payoffs are equal to

$$p(H)\tau^*[v(H) - v(L)]p(L) \text{ for bidder 1,}$$

$$\text{and} \quad 0 \text{ for bidder 2.}$$

2 Price trends in sequential auctions

Since the expected prices fetched in a first-price auction represent the seller's expected revenue and since the latter is determined by the difference between the object's expected value to the seller and the sum of the bidders' *ex ante* expected payoffs, there are three cases to consider.

First, if $\sigma < \frac{p(L)}{p(L)+1}$, then bidder 1 always conceals her signal in the first stage by bidding as if she is Low-signalled, i.e., $a = \sigma v(L)$, and she bids for the second object as she would for the "second" object in a simultaneous auction, by drawing a bid from F_{1H}^2. As for bidder 2, she optimally bids $\sigma v(L)$ for the first object and draws her second-stage bid from F_2^2. Thereore, using (3), the first and second stage expected prices are equal to

$$\begin{aligned} P_1^C &= \sigma v(L), \\ P_2^C &= (1-\sigma)\left\{E(V) - p(H)p(L)[v(H) - v(L)]\right\}. \end{aligned} \tag{14}$$

Taking the difference $\Delta_P^C = P_2^C - P_1^C$ yields $E(V) - p(H)p(L)(1-\sigma)[v(H) - v(L)] - \sigma v(L)$, which is always positive for $\sigma < \frac{p(L)}{p(L)+1}$ so expected prices are always increasing when bidder 1 conceals her signal in the first stage.

Second, if $\frac{p(L)}{p(L)+1} < \sigma < \frac{1}{p(L)+1}$, then bidder 1 randomizes between revealing and concealing her High-signal in the first stage. Therefore, using (13), and recalling that bidder 1 conceals her High signal with probability $G_{1H}(\sigma v(L))$, the expected prices of the first and second object are equal to

$$\begin{aligned} P_1^R &= \sigma E(V) - [v(H) - v(L)]\left\{\left[2\sqrt{(1-\sigma)\sigma p(L)} - p(L)\right] - \sigma p(L)\right\}, \\ P_2^R &= (1-\sigma)E(V) - p(L)[v(H) - v(L)]\left[1 - \sigma - \sqrt{(1-\sigma)\sigma p(L)}\right]. \end{aligned} \tag{15}$$

It then follows that $\Delta_P^R = P_2^R - P_1^R$ is equal to

$$E(V)(1-2\sigma) + [v(H) - v(L)] \left\{ 2\sqrt{(1-\sigma)\sigma p(L)} - p(L) \left[2 - \sigma - \sqrt{(1-\sigma)\sigma p(L)} \right] \right\},$$

which is always positive for $\sigma = \frac{p(L)}{p(L)+1}$ and always negative for $\sigma = \frac{1}{p(L)+1}$ so that price trends should be decreasing (increasing) beyond (below) some critical value of σ in $\left(\frac{p(L)}{p(L)+1}, \frac{1}{p(L)+1} \right)$. Figure 3 displays the (p, θ, σ)-constellations for which price trends are decreasing, constant or increasing. The region delimited by the hyperplanes $\sigma = \frac{p(L)}{p(L)+1}$ (in red) and $\Delta_P^R = 0$ (in gray) characterises the (p, θ, σ)-values for which price trends are increasing ($\Delta_P^R > 0$) whereas the region between the hyperplanes $\Delta_P^R = 0$ and $\sigma = \frac{1}{p(L)+1}$ (in blue) characterises those for which price trends are decreasing ($\Delta_P^R < 0$). Notice that the hyperplane $\sigma = 0.5$ lies below the one for $\Delta_P^R = 0$ for all values of p and θ so expected prices always increase when the objects are identical.

< Figure 3 about here >

Third, if $\sigma > \frac{1}{p(L)+1}$, then bidder 1 always reveals her signal in the first stage by bidding as she would for the "first" object in a simultaneous auction (see Appendix I). In this case, bidder 2 invariably draws her bid for the first stage from F_2^1, and bids $v(H)$ in the second stage if bidder 1 drew a bid from F_{1H}^1 in the first stage and $v(L)$ if not (see Appendix II). Therefore, the expected prices in the first and second stages are equal to

$$\begin{aligned} P_1^{\neg C} &= \sigma \left\{ E(V) - p(L)p(H)[v(H) - v(L)] \right\} \\ P_2^{\neg C} &= (1-\sigma)E(V) \end{aligned} \tag{16}$$

Taking the difference $\Delta_P^{\neg C} = P_2^{\neg C} - P_1^{\neg C}$ yields $(1-2\sigma)E(V) + \sigma p(L)p(H)[v(H) - v(L)]$ which is always negative for $\sigma > \frac{1}{p(L)+1}$ so price trends are always decreasing when bidder 1 reveals her signal in the first stage.

3 Expected revenue comparisons

3.1 Indivisible objects

The seller's expected revenues can directly be determined from the expected prices. For the simultaneous format, it is determined from (16) and (14) by observing that $P_1^{\neg C}$ corresponds to the expected price of the "first" object and P_2^C corresponds to that of the "second" object, so

$$E(\text{Rev}_{Sim}) = E(V) - p(H)p(L)[v(H) - v(L)]. \tag{17}$$

For the sequential format, given bidder 1's possible strategies for the first stage, there are three cases to consider. First, if bidder 1 always conceals her signal, then it follows from (14) that the seller's expected revenue is equal to

$$E(\text{Rev}_{Seq}^C) = \sigma v(L) + (1-\sigma)\left\{E(V) - p(H)p(L)[v(H) - v(L)]\right\},$$

so her preferred format is determined by

$$\Delta_R^C = E(\text{Rev}_{Sim}) - E(\text{Rev}_{Seq}^C) = \sigma\left\{E(V) - p(H)p(L)[v(H) - v(L)] - v(L)\right\},$$

which is always positive so the seller prefers the simultaneous format when bidder 1 always conceals her High-signal. Note that this is opposite to Hausch (1986)'s result for symmetric bidders and identical objects.

Second, if bidder 1 randomises between concealing and revealing her signal, then it follows from (15) that

$$E(\text{Rev}_{Seq}^R) = E(V) + [v(H) - v(L)]\left\{p(L)[\sigma + \sqrt{(1-\sigma)\sigma p(L)}] - 2\sqrt{(1-\sigma)\sigma p(L)}\right\}.$$

Note that changing the order of sales in this framework simply reverts to defining $\xi = 1 - \sigma$ for the "new" first object, i.e., the second object before the order reversal. By implementing the change in the sales' order, the seller's expected revenue would now be equal to

$$E(\text{Rev}_{Seq}^{R^*}) = E(V) + [v(H) - v(L)]\left\{p(L)[1 - \sigma + \sqrt{\sigma(1-\sigma)p(L)}] - 2\sqrt{\sigma(1-\sigma)p(L)}\right\}.$$

which is greater than $E(\text{Rev}_{Seq}^R)$ for $\sigma \in \left(\frac{p(L)}{p(L)+1}, 0.5\right)$, and smaller for $\sigma \in \left(0.5, \frac{1}{p(L)+1}\right)$. Thus, when bidder 1 randomises between revealing and concealing her signal, the seller is always better off selling the most valuable object first. As for the seller's preferred format *before* a possible order reversal, it is determined by

$$\Delta_R^R = E(\text{Rev}_{Sim}) - E(\text{Rev}_{Seq}^R)$$
$$= [v(H) - v(L)] \left\{ 2\sqrt{\sigma p(L)} - p(L) \left[p(H)(1+\sigma) - \sigma - \sqrt{\sigma p(L)} \right] \right\},$$

which can either be positive or negative, depending on the auction's specifics. Note that for $\sigma \in \left(\frac{p(L)}{p(L)+1}, 0.5\right)$, changing the order of sales yields $\Delta_R^{R*} = E(\text{Rev}_{Sim}) - E(\text{Rev}_{Seq}^{R^*}) < \Delta_R^R$, i.e., a smaller revenue difference between the two formats, and may lead the seller to change his initial preference for a simultaneous auction into a preference for a sequential auction.

Figures 4(a) and 4(b) display $\Delta_R^R = 0$ in terms of (p, θ) when $\sigma = 0.5$ or $\sigma = 0.6$, assuming $\underline{V} = 0$ and $\overline{V} = 10$. When the objects are identical ($\sigma = 0.5$), the simultaneous and sequential formats are revenue equivalent when $\theta = .3904$ for $p = 1$ and $0 < \theta < 1$, as reported by Engelbrecht-Wiggans and Weber (1983) for $k = 2$. The regions above (below) the zero-difference hyperplanes characterize the (p, θ)-constellations for which the simultaneous (sequential) format generates higher expected revenues. The figures thus indicate that simultaneous auctions are mostly preferred when the objects are likely to be of high value. Figure 4(c) reports $\Delta_R^R = 0$ when $\sigma = 0.25$ (in yellow) and Δ_R^{R*} (in blue). Note that Δ_R^R in Figure 4(b) and Δ_R^R and Δ_R^{R*} in Figure 4(c) account for the (p, θ)-constellations where bidder 1 will reveal her signal — i.e., for low values of θ and high values of p, c.f. Figure 3. Figure 4(c) indicates that the simultaneous format is unilaterally preferred to the sequential one when $\sigma = 0.25$ and that it is much less so once the order of sale is changed.

< Figure 4 about here >

Third, if bidder 1 always reveals her signal, then it follows from (16) that

$$E(\text{Rev}_{Seq}^{\neg C}) = E(V) - \sigma p(L) p(H)[v(H) - v(L)] \tag{18}$$

and $\Delta_R^{\neg C} = E(\text{Rev}_{Sim}) - E(\text{Rev}_{Seq}^{\neg C}) = (\sigma - 1) p(H) p(L)[v(H) - v(L)]$ which is always negative so the seller prefers the sequential format when bidder 1 reveals her signal. Furthermore, since $E(\text{Rev}_{Seq}^{\neg C}) > E(\text{Rev}_{Seq}^R) > E(\text{Rev}_{Seq}^C)$, it follows that for a given (p, θ, σ)-constellation such that bidder 1 would conceal her High signal (i.e., when $\sigma \leq \frac{p(L)}{p(L)+1}$), the seller could achieve her highest possible expected revenue by changing the order of sales as that would lead to $\tau \geq \frac{1}{p(L)+1}$; a situation where bidder 1 would reveal her signal in the first stage.

3.2 Divisible object

It follows from the above that auctioning a divisible object simultaneously reverts to selling all the produce in the first stage of a sequential auction or equivalently, setting $\tau = 1$. The seller's expected revenue is thus equal to:

$$E(\text{Rev}_{Sim}^{D}) = E(V) - p(L)p(H)[v(H) - v(L)]$$

By selling sequentially and ensuring that bidder 1 reveals her signal in the first stage, i.e., choosing $\tau \in (\frac{1}{1+p(L)}, 1]$ it follows from (18) that the seller's expected revenue is then equal to

$$E(\text{Rev}_{Seq}^{D}) = E(V) - \tau p(L)p(H)[v(H) - v(L)]$$

which is maximised when τ takes its smallest possible value in the range $(\frac{1}{1+p(L)}, 1]$. In this case, $\Delta_R^D = E(\text{Rev}_{Sim}^D) - E(\text{Rev}_{Seq}^D) = (\tau - 1)p(L)p(H)[v(H) - v(L)]$ which is negative so the seller always prefers the sequential format when the asset for sale is divisible.

4 Conclusion

This paper compares the relative performance of simultaneous and sequential first-price auctions of non-identical objects in the context of one-sided incomplete information and common values. The analysis provides the following main results. First, selling the most valuable object first in a sequential auction maximises the seller's expected revenues. Second, the more the objects are different and the more the sequential format favours the informed bidder. Third, by switching the order of sales, the seller may be led to change her initial preference for a simultaneous format (in which bidders submit object-specific bids) to one for a sequential format. Fourth, sequential auctions are mostly preferred by the seller when the objects are likely to be of low value and the precision of the informed bidder's signal is low. Finally, if the asset to be sold is perfectly divisible, then the seller is better off designing a sequential auction such that the informed bidder is induced to reveal her signal in the first stage; this allowing the seller to extract all the bidders' surplus in the second stage.

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Appendix I: Proofs

Simultaneous format

Equilibrium

Suppose that (the uninformed) bidder 2 uses her prescribed strategy and draws her bid for the first object from F_2^1, which is defined on $[\sigma v(L), \bar{b}^1]$. A Low-signalled bidder 1 can then bid:

1. $b < \sigma v(L)$ but will never win so her payoff equals 0.
2. $b = \sigma v(L)$. She will then either tie with bidder 2 (with probability $p(L)$) or lose the auction. In both cases, she earns a zero expected payoff.
3. $b \in (\sigma v(L), \bar{b}^1]$ and earn an expected payoff of $(\sigma v(L) - b)F_2^1(b) < 0$ whenever she wins.
4. $b > \bar{b}^1$ and will always win and make a negative payoff of $\sigma v(L) - b$.

It thus follows that the best-response for a Low-signalled bidder 1 is to bid $b = \sigma v(L)$.

If bidder 1 is High-signalled, then she can bid:

1. $b < \sigma v(L)$ but will never win so her payoff equals 0.
2. $b = \sigma v(L)$. She will then either tie with bidder 2 (with probability $p(L)$) or lose the auction. In both cases, she earns a zero expected payoff.
3. $b \in (\sigma v(L), \bar{b}^1]$ and earn an an expected payoff of $(v(H) - b)F_2(b) = [v(H) - v(L)]p(L) > 0$.
4. $b > \bar{b}^1$ and will always win and make a negative payoff of $v(H) - b$.

It thus follows that any bid in $(\sigma v(L), \bar{b}^1]$ generates the same positive expected payoff and that $F_{1H}^1(b)$ is a best-response.

Suppose now that (the informed) bidder 1 uses her prescribed strategy and bids $\sigma v(L)$ upon receiving a Low-signal and that she draws a bid from $F_{1H}^1(b)$ upon receiving a High signal. Bidder 2 (who receives no signal) can then bid:

1. $b < \sigma v(L)$ but will never win so her payoff equals 0.

2. $b \in [\sigma v(L), \bar{b}^1]$. She will either tie with a Low-signaled bidder 1 and earn zero payoff or she will win with probability $F_{1H}^1(b)$ and earn an expected payoff of $(\sigma v(H) - b)p(H)F_{1H}^1(b) + (\sigma v(H) - b)p(L) = 0$.

3. $b > \bar{b}^1$ and will always win and make a negative payoff of $\sigma [p(L)v(L) + p(H)v(H)] - b$.

It thus follows that any bid in $[\sigma v(L), \bar{b}^1]$ generates the same positive expected payoff and that $F_2(b)$ is a best-response.

It also follows from the above that the expressions of the bidders' equilibrium distributions for the second object then take the following expressions:

$$F_{1H}^2(b) = \frac{[b - (1-\sigma)v(L)]p(L)}{[(1-\sigma)v(H) - b]p(H)} \quad \text{with } b \in ((1-\sigma)v(L), \bar{b}^2],$$

$$\text{and} \quad F_2^2(b) = \frac{(1-\sigma)[v(H) - v(L)]p(L)}{(1-\sigma)v(H) - b} \quad \text{with } b \in [(1-\sigma)v(L), \bar{b}^2].$$

Sequential format

It first needs to be shown that the equilibrium for this format has one of the following three features: *(i)* A High-signalled bidder 1 always reveals her signal in the first auction, *(ii)* A High-signalled bidder 1 always conceals her signal in the first auction, or *(iii)* A High-signalled bidder 1 randomly reveals her signal in the first auction.

Always revealing: Upon always revealing her High-signal, bidder 1's information would become common knowledge and both bidders would consequently bid the second object's expected value $(1-\sigma)v(H)$, leading to zero expected profits in the second stage. Therefore, if bidder 1 always reveals her High-signal, then the equilibrium for the first stage has the same form as the one for the "first" object in a simulatenous sale; with (the High-signalled) bidder 1 earning an expected profit of $\sigma[v(H) - v(L)]p(L)$, and bidder 2 earning zero expected profits. Note that it follows from the proof for the simultaneous format that if a High-signalled bidder 1 draws her a bid from $F_{1H}^1(b)$ with $\sigma = 1$ if she is High-signalled and bids $\sigma v(L)$ upon receiving a Low signal, then bidder 2 can do no better than to draw a bid from $F_2^1(b)$.

Now assume that a High-signalled bidder 1 considers bidding $\sigma v(L)$ in the first stage so as to deceive bidder 2 into thinking that she is Low-signalled. This deviation would generate an expected payoff of 0+ $(1-\sigma)[v(H) - v(L)]$. The first part stands for the fact she always loses the first stage and the second part stands for the expected payoff from winning the second object, having anticipated bidder 2's bid of $(1-\sigma)v(L)$ and submitting an infinitesimally higher bid. Thus, for bidder 1 to always reveal her High-signal, it must be that $\sigma[v(H)-v(L)]p(L) > (1-\sigma)[v(H)-v(L)]$, or equivalently that $\sigma > \frac{1}{p(L)+1}$. A High-signalled bidder 1 would thus find it worth to reveal her signal only if the first object is of sufficiently higher value than the second.

Always concealing: Upon always concealing her High-signal by bidding as if she is Low-signalled, the first stage delivers no information so bidding for the second stage would be like bidding for the "second" object in a simultaneous sale. If bidder 1 uses this strategy, then bidder 2 always wins the tie with a bid of $\sigma v(L)$ for the first object and she can do no better than to bid for the second stage as if she bids for the "second" object in a siimultaneous sale. In this case, the expected payoffs would be $0+(1-\sigma)[v(H)-v(L)]p(L)$ for a High-signalled bidder 1 and $\sigma[v(H)-v(L)]p(H)+0$ for bidder 2.

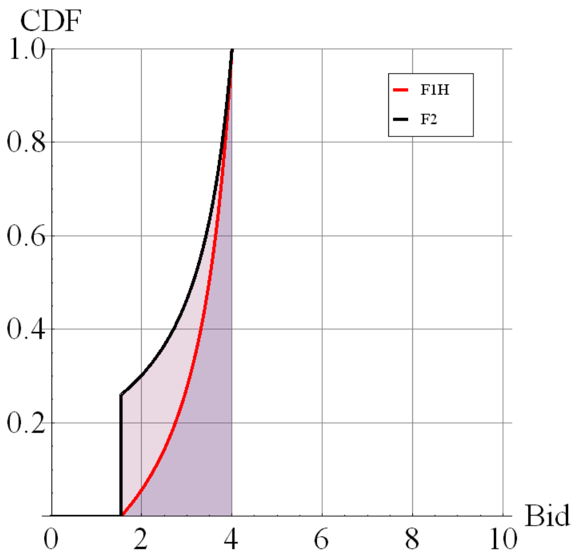
Now assume that a High-signalled bidder 1 considers bidding an infinitesimally higher bid than $\sigma v(L)$, thereby winning the first object for sure and revealing her High signal. This deviation would generate an expected payoff over the two auctions of $\sigma[v(H)-v(L)]$ for bidder 1 and of 0 for bidder 2. Thus, for bidder 1 to always conceal her High-signal in the first stage to be an equilibrium, it must be that $\sigma\,[v(H)-v(L)] < (1-\sigma)[v(H)-v(L)]p(L)$, or equivalently that $\sigma < \frac{p(L)}{1+p(L)}$. A High-signalled bidder 1 would thus find it worth to conceal her signal only if the first object is of sufficiently lower value than the second.

Randomising: It follows from the above that for $\frac{p(L)}{p(L)+1} < \sigma < \frac{1}{p(L)+1}$, bidder 1 will neither "always reveal" nor "always conceal" her High signal and will thus randomize between these two alternatives.

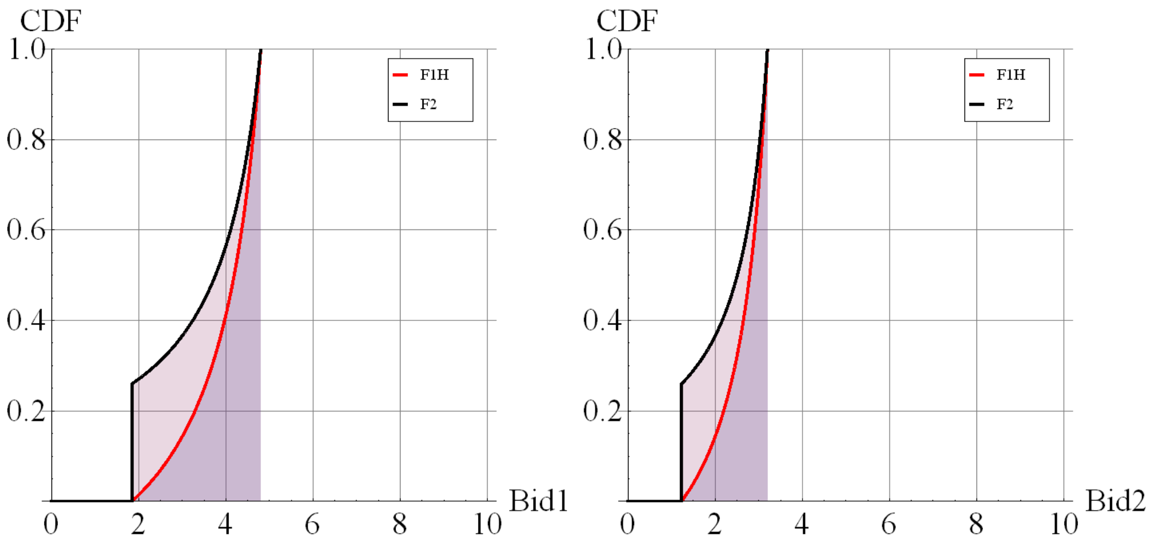
FIGURES

Figure 1: Equilibrium Bid Distributions for Simultaneous Auctions

when $V_{max} = 10$ with probability 0.8 and $V_{min} = 0$ otherwise, and $p = 0.9$.



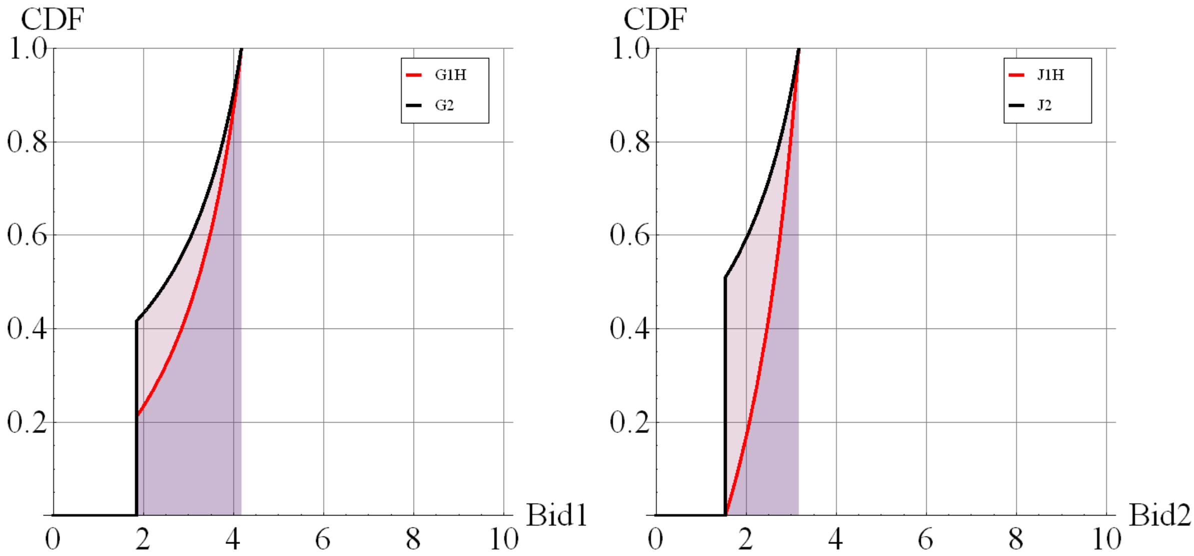
$\sigma = 0.5$ [Identical objects]



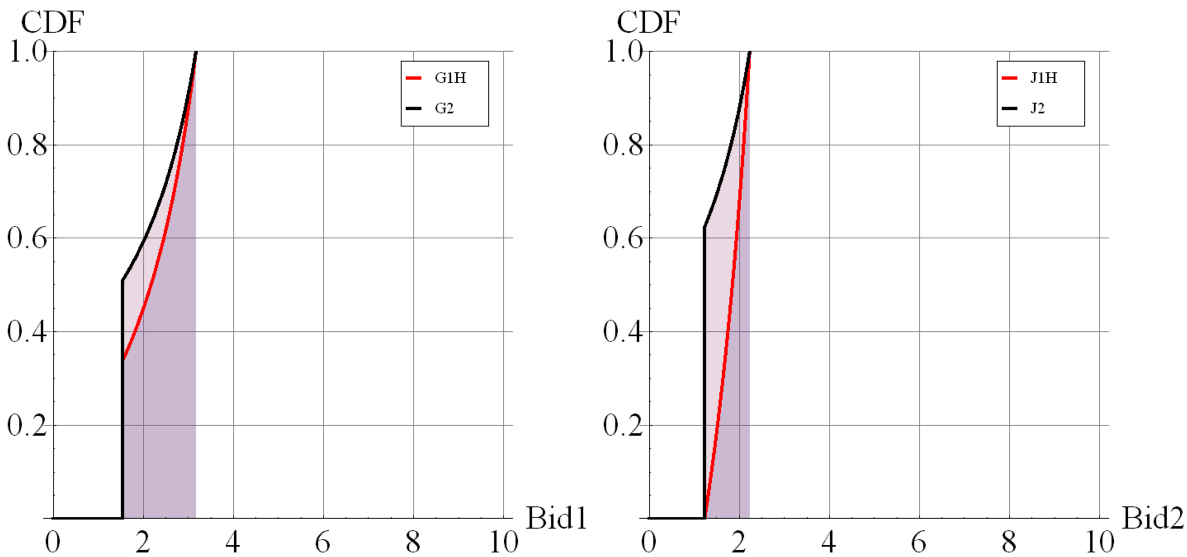
$\sigma = 0.6$ [1st object more valuable]

Figure 2: Equilibrium Bid Distributions for Sequential Auctions

when $V_{max} = 10$ with probability 0.8 and $V_{min} = 0$ otherwise, and $p = 0.9$.



σ = 0.5 [Identical objects]



σ = 0.6 [1st object more valuable]

Figure 3: Equilibrium Price Trends

when $V_{max} = 10$ and $V_{min} = 0$. [p : signal precision, θ: probability that $V_{max} = 10$]

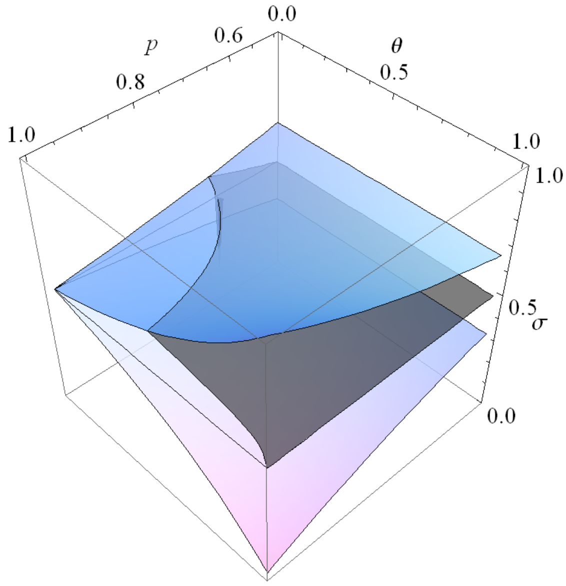


Figure 4(a): Expected Revenue Difference (Δ = Simultaneous – Sequential) <u>when σ = 0.5</u>

when $V_{max} = 10$ and $V_{min} = 0$. [p : signal precision, θ: probability that $V_{max} = 10$]

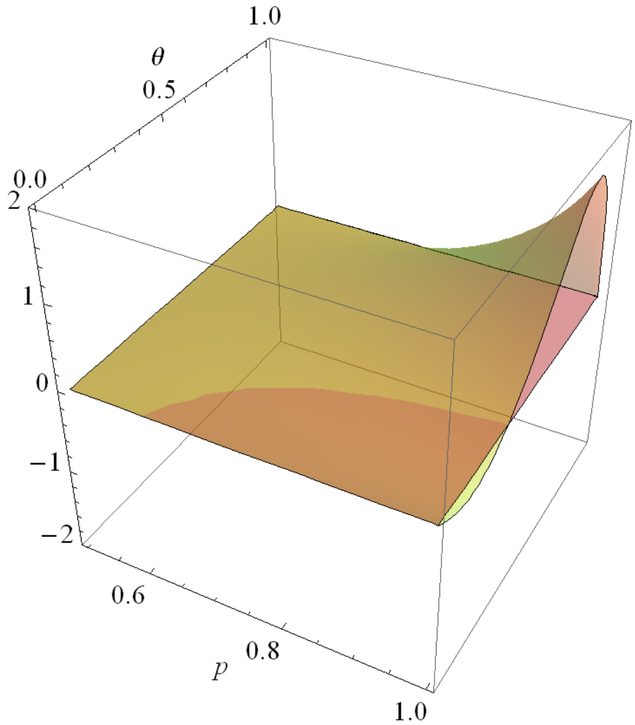


Figure 4(b): Expected Revenue Difference (Δ = Simultaneous – Sequential) <u>when σ = 0.6</u>

when $V_{max} = 10$ and $V_{min} = 0$. [p : signal precision, θ: probability that $V_{max} = 10$]

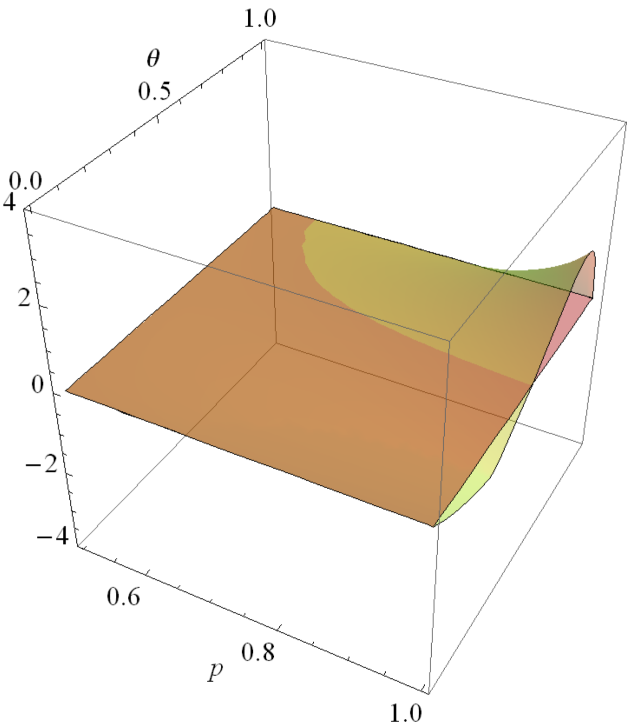
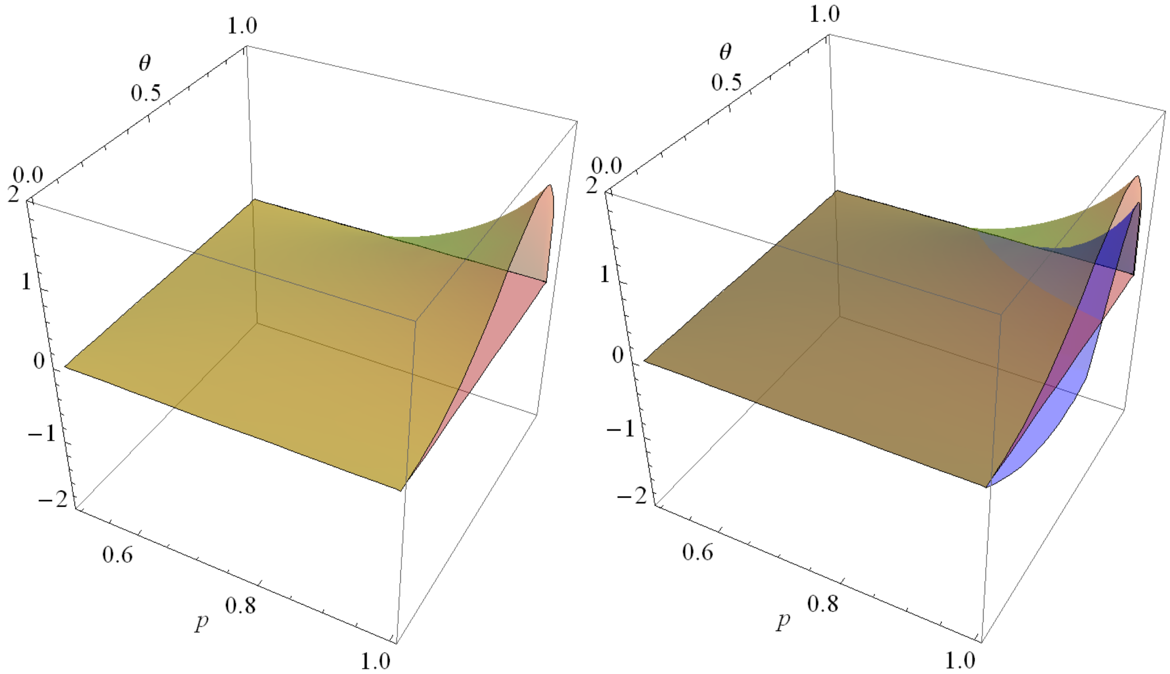


Figure 4(c): Expected Revenue Difference (Δ = Simultaneous – Sequential) <u>when σ = 0.25</u>

when $V_{max} = 10$ and $V_{min} = 0$. [p : signal precision, θ: probability that $V_{max} = 10$]



Before Reversing order of sales **After** reversing the order of sales