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Working Papers

ISSN 2203-6024

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Working Paper No. 2015-13
April 2015

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Believing in correlated types in spite of independence: An indirect evolutionary analysis

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Abstract: The risk-neutral equilibrium bidding strategy for first-price auctions with independent private values is justified without assuming a well-defined Bayesian game. Bidders, aware of their own value, assume the private values to be linearly related. The latter, however, are independent and identically distributed, and this is only known by Nature. Allowing for arbitrary linear common value beliefs, and assuming optimal bidding for such beliefs we derive the unique evolutionarily stable conjectural belief and justify risk neutral bidding in a new and hopefully innovative way.

Keywords: Evolutionarily stable strategy, indirect evolution, first-price auction, independent private values, symmetric Bayes-Nash equilibrium.

J.E.L. Classifications: C7, D4, D8.

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1. Introduction

Strategic interaction of privately informed players is traditionally analyzed by assuming a commonly known prior representing others' beliefs about own private information (Harsanyi, 1967a, 1967b, 1968). Thus, private information, or more generally incomplete information, is transformed into imperfect information about partially unobservable chance moves. And since these fictitious chance moves are commonly known, the game is informationally closed, i.e., one considers a Bayesian game.

One justifies such transformation of incomplete information into imperfect information by common knowledge of rationality which induced Harsanyi (1967/8) to suggest even consistent priors, i.e., all players agree on how each player's private information is randomly generated. As a consequence, each player is supposed to engage in a very demanding counterfactual exercise: even though each player knows his own type, he chooses how to decide if he would be of any other type expected by his co-players.

In this note, we provide an alternative justification of risk neutral equilibrium bidding for the first-price auction with independent private values (Vickrey, 1961). Bidders react optimally to their deterministic conjectural beliefs about others but by entertaining such beliefs, they are subject to evolutionary selection. Our indirect evolutionary approach avoids common knowledge of priors and rationality as well as counterfactual reasoning.

In spite of their independent private values, bidders entertain conjectural common value beliefs according to which they infer the unknown value of the other bidder from their own value, i.e., they believe to be bidding in a first-price auction with complete information. This, of course, means that competing bidders entertain inconsistent conjectural beliefs.

How the true prior randomly generates private values is only known by Nature which selects evolutionarily among conjectural beliefs. Assuming that bidders' expected profits measure their fitness, i.e., (in evolutionary biology terminology) reproductive success, the unique evolutionarily stable common-value belief can be derived. It suggests for a particular family of probability distributions (priors), the same equilibrium bidding behavior as the orthodox equilibrium analysis of the first-price auction game with independent private values.

2. Analysis

Before providing the analytic details let us comment on indirect evolution, which combines rationality in decision making and evolution of the rules (of the game). In our example, the evolving rules are the individual conjectural beliefs about others' evaluations which determine "decision utility" and thus the optimal behavior. This in turn, yields the fitness of such rules, respectively conjectural beliefs, measured by the true (expected) profits and allows the derivation of rules, respectively conjectural beliefs, which are evolutionarily stable. Whereas in direct evolution behavior is directly selected according to fitness, here evolution selects among rules, specifically beliefs, to which one then responds optimally.

Let $i = 1, \dots., n$ denote n randomly matched bidders of an infinite population of bidders with values denoted by $v_i \in \mathbb{R}_+$. Only Nature is aware that for $i = 1, \dots, n$, the n values v_i are independently and identically distributed according to a probability distribution F with density f defined on $\mathbb{R}_+$.

Each bidder i, aware of his own value v_i, assumes that the highest value of the $n-1$ other values, v_j, is linearly related to his own via $v_j = q_i v_i$ with $0 \leq q_i \leq 1$ and that is also what each of his $n-1$ competitors believes. Thus bidder i views the bidding contest as one with complete information about the two evaluations, namely the own value v_i and the highest of the $n-1$ other values $v_j = q_i v_i$.[2] This conjecturally perceived bidding contest with complete information in view of bidder i has (for $q_i < 1$) multiple equilibria but only one in weakly undominated strategies. To save on technical notation assume that i wins in case of equal bids: since bids $b_j > q_i v_i$ are seen as weakly dominated, the unique solution is $b_i\,(v_i) = q_i v_i = b_j$.[3,4] Thus, each bidder $i = 1, \dots, n$ believes to earn $(1 - q_i)v_i$ for all $v_i \in \mathbb{R}_+$. However, such conjectural payoff has to be distinguished from a bidder's expected profit which we assume to measure reproductive success. Hereafter, we derive expected profits and justify them as measuring fitness.

Consider a q-monomorphic belief population with $0 < q < 1$ and assume a mutant p-belief type with value v and $0 \leq p \leq 1$ which might invade the population. The expected payoff of such mutant belief p is given by:

[2] This resembles the justification of monopolistic competition by Güth and Huck (1997) who allow for conjectural beliefs concerning the interdependency of competitors and show that believing to be independent of other's sales behavior is evolutionarily stable (see also Güth, 1998).

[3] More precisely, i expects to earn $v_i - b_i$ as long as $b_i \geq b_j$ and nothing otherwise. Bidder j $(\neq i)$, too, is aware of v_i as well as of $v_j = q_i v_i$ and, since also rational, avoids overbidding $(b > v_j)$. Thus, in view of common(ly known) rationality, the solution requires bids $b_i = q_i v_i = b_j$.

[4] The first-price auction format is not incentive compatible but is overbidding proof.

$$\int_{pv>qv_i}(v-pv)dF(v_i)^{n-1}=(1-p)v\int_0^{\frac{p}{q}v}(n-1)F(v_i)^{n-2}f(v_i)dv_i$$

for all $v\in\mathbb{R}_+$. To justify expected profit as fitness requires two assumptions: one being that in real-world markets, commercial survival depends on profits, a familiar idea in economics. The other assumption is employed to neglect the stochastic nature of profits. We rely on the typical idea (in evolutionary theory) of an infinite population with the random formation of groups of size n. If each bidder is engaged in sufficiently many such encounters, then the variance of average profits becomes negligible and selection is deterministically governed by expected profits.

For the evolutionarily stable monomorphic belief $q^*>0$ it is necessary that $p=q^*$ is a best reply to q^*, i.e., $p=q^*$ solves

$$\operatorname*{Max}_{p\in[0,1]}\quad(1-p)v\int_0^{\frac{p}{q}v}dF(v_i)^{n-1}$$

The first-order condition for maximization with respect to p due to $v>0$ is:

$$-\int_0^{\frac{p}{q}v}dF(v_i)^{n-1}+(1-p)(n-1)F\left(\frac{p}{q}v\right)^{n-2}f\left(\frac{p}{q}v\right)\frac{v}{q}=0$$

Imposing symmetry, i.e., $p=q^*$, and solving for q^* yields:

$$-\int_0^{v}dF(v_i)^{n-1}+(1-q^*)(n-1)F(v)^{n-2}f(v)\frac{v}{q^*}=0$$

$$q^*=\frac{(n-1)vf(v)F(v)^{n-2}}{(n-1)vf(v)F(v)^{n-2}+\int_0^v dF(v_i)^{n-1}}$$

We now compare this to the symmetric Bayes-Nash equilibrium strategy for the independent private values first-price auction with n bidders and F having density f defined on $\mathbb{R}_+$ (Vickrey, 1961):

$$b(v)=v-\frac{\int_0^v F(t)^{n-1}dt}{F(v)^{n-1}}$$

or equivalently (since $v\in\mathbb{R}_+$ and thus $v>0$):

$$b(v) = \left(1 - \frac{\int_0^v F(t)^{n-1} dt}{vF(v)^{n-1}}\right) v$$

Thus, for our equivalence result to obtain, all what is required is that

$$q^* \equiv \frac{(n-1)vf(v)F(v)^{n-2}}{(n-1)vf(v)F(v)^{n-2} + \int_0^v dF(v_i)^{n-1}} = 1 - \frac{\int_0^v F(t)^{n-1} dt}{vF(v)^{n-1}}$$

Thus the only evolutionarily stable conjectural belief q^* in the considered class of linear belief types justifies equilibrium bidding if the above equation is satisfied.

This condition holds in particular for the family of Beta distributions $B(v, \alpha, 1)$ with density $f(v) = v^{\alpha-1}/B(\alpha, 1)$ defined on $(0,1)$, with $\alpha > 0$ and $B(\alpha, 1) = \Gamma(\alpha)\Gamma(1)/\Gamma(\alpha+1)$. For such distributions, we get $q^* = \alpha(n-1)/[1 + \alpha(n-1)]$.[5]

Since the best reply is unique, the second requirement for an evolutionarily stable strategy (see Maynard Smith, 1982) does not have to be considered.[6] Thus after evolution has taken its course, all bidders i with value v_i of an infinite population would have had an infinity of encounters with $n-1$ bidders and will believe that the highest value of their $n-1$ competitors is $q^* v_i$, which leads them to bid $b_i(v_i) = q^* v_i$ for $v_i \in \mathbb{R}_+$.

The result yields two messages, a basic one as well as a rather special result. The basic message is that we derive bid functions in a very different way from orthodox equilibrium analysis of Bayesian games. We do not assume a commonly known prior, but allow bidders to deny incomplete information by entertaining common value beliefs which, however, are subjected to evolutionary selection. The special result is that this completely different approach can nevertheless suggest the same bid functions, namely for a particular class of density functions.

2. Discussion

The analysis above does not require any kind of common knowledge but does not deny rationality altogether. Having no clue how the private values are generated, bidders simply believe that the other bidders will evaluate the commodities proportionally as they do but

[5] This simply follows from the fact that the symmetric risk neutral Bayes-Nash equilibrium bidding strategy is linear in values if these are drawn from a $B(\alpha, 1)$ distribution. See Pezanis-Christou and Romeu (2007) for a structural analysis of first-price auction data that uses this property.

[6] This second requirement is that an alternative best reply $q \neq q^*$ fares worst than q^* in the q-monomorphic population.

focus on the case of appreciating commodities more than the other. Given such mutually inconsistent conjectural beliefs, they bid rationally (i.e., bids conform to the unique perfect equilibrium of their individually perceived first-price auction with complete information).

Thus like many studies of so-called indirect evolution (see the selective review of Berninghaus, Güth and Kliemt, 2012), we allow for rationality, based on decision utility, as well as for evolution and thus path dependence, based on fitness as often, in economic applications, measured by profits. Our basic message is a conceptually very different way of deriving bid functions, here for the first-price auction. Surprisingly, such common value beliefs, whose evolution is guided by expected profits, can lead after evolutionary selection to the same bidding behavior as suggested by orthodox game theoretic reasoning.

Acknowledgements: We thank an anonymous referee for helpful comments. Financial support from the Max Planck Institute and from the Australian Research Council through a *Discovery Project* grant (DP140102949) is gratefully acknowledged.

References

Berninghaus S., W. Güth and H. Kliemt, 2012, "Pull, push or both: indirect evolution in economics and beyond". In *Evolution and rationality: decisions, co-operation and strategic behavior*. Eds. Okasha S. and K. Binmore. Cambridge University Press, Cambridge.

Güth W., 1998, "Are rational cost expectations evolutionarily stable?", IFO-Studien: Zeitschrift für empirische Wirtschaftsforschung, 44(1), 1-13.

Güth W. and Huck, S., 1997, "A new justification of monopolistic competition", *Economics Letters*, 57(2), 177-182.

Harsanyi J., 1967, "Games with incomplete information played by 'Bayesian' players, I-III. Part I. The basic model", *Management Science*, 14(3), 159-182.

Harsanyi J., 1968a, "Games with incomplete information played by 'Bayesian' players, I-III. Part II. Bayesian equilibrium points", *Management Science*, 14(5), 320-334.

Harsanyi J., 1968b, "Games with incomplete information played by 'Bayesian' players, I-III. Part III. The basic probability distribution of the game", *Management Science*, 14(7), 486-502.

Maynard Smith J., *Evolution and the Theory of Games*, Cambridge: Cambridge University Press, 1982.

Pezanis-Christou P. and A. Romeu Santana, 2007, "Structural inferences from first-price auction data", *mimeo,* University of Strasbourg.

Vickrey W., 1961, "Counterspeculation, auctions, and competitive sealed tenders", *Journal of Finance*, 16(1), 8-37.