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Firmin Doko Tchatoka
Nicolas Groshenny Qazi
Haque
Mark Weder

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Monetary Policy and Indeterminacy after the 2001 Slump*

Firmin Doko Tchatoka† Nicolas Groshenny Qazi Haque
Mark Weder‡

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Abstract

This paper estimates a New Keynesian model of the U.S. economy over the period following the 2001 slump, a period for which the adequacy of monetary policy is intensely debated. To relate to this debate, we consider three alternative empirical inflation indicators in the estimation. When using CPI or PCE, we find support for the view that the Federal Reserve's policy was extra easy and may have led to equilibrium indeterminacy. The interpretation changes when using core PCE and monetary policy appears to have been reasonable and sufficiently active to rule out indeterminacy. We then relax the assumption that inflation in the model is measured by a single indicator. We re-formulate the artificial economy as a factor model where the theory's concept of inflation is the common factor to the three empirical inflation series. We find that CPI and PCE provide better indicators of the latent concept while core PCE is less informative. Again, this procedure cannot dismiss indeterminacy.

*JEL codes **E32**, **E52**, **E58**. *Keywords*: Great Deviation, Indeterminacy, Taylor Rules.

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‡Corresponding author (mark.weder@adelaide.edu.au).

1 Introduction

The Great Recession was the deepest recession in U.S. economic activity in the post-war era. What caused this massive macroeconomic contraction? As one of the key figures in the debate, Taylor (2007, 2012) blames inept monetary policy. In particular, he asserts that the Federal Reserve kept the policy rate *too low for too long* following the 2001 slump. He argues that this loose policy contributed to a housing boom and that it was this environment that ultimately brought the economy close to the brink. To bolster his thesis of an extra easy monetary policy, Taylor constructs an artificial path for the Federal Funds rate that follows his proposed rule. He characterizes this counterfactual rate's loose fitting to the actual rate as

> "[...] the biggest deviation, comparable to the turbulent 1970s." [Taylor, 2007, 2]

This view is disputed by many. Amongst them, Bernanke (2010) argues that Taylor's use of the headline consumer price index (CPI) to measure inflation in the Federal Reserve's reaction function is misleading. In fact, the Federal Reserve switched the inflation measures that inform its monetary policy deliberations several times over the last two decades. In particular, it moved away from the CPI to the personal consumption expenditure deflator (PCE) in early 2000. In turn, PCE was abandoned midway through 2004 in favor of the core PCE deflator (which excludes food and energy prices).[1] Bernanke (2015) revisits Taylor's exercise and constructs his own counterfactual Federal Funds rate using core PCE. Bernanke's verdict of the Federal Reserve's policy during the 2000s is inimical to Taylor's and he says that

> "[...] the predictions of my updated Taylor rule and actual Fed policy are generally quite close over the past two decades. In particular, it is no longer the case that the actual funds rate falls below the predictions of the rule in 2003-2005." [Bernanke, 2015]

Our paper sheds further light on this debate. It takes as a point of departure Taylor's claim of an analogy of the 1970s and the 2000s as well as one of the key policy

[1] See Mehra and Sawhney (2010).

recommendations for monetary policy that has emanated from the New Keynesian modelling: interest rates should react strongly to inflation movements to not destabilize the economy. Phrased alternatively, if the central bank's response to inflation is tuned too passively in a Taylor rule sense, multiplicity and endogenous instability may arise. In fact, the U.S. economy of the 1970s can be well represented by an indeterminate version of the New Keynesian model as was shown by Lubik and Schorfheide (2004). Along these lines, the current paper turns Taylor's *too low for too long* story into questioning whether the Federal Reserve operated on the indeterminacy side of the rule after the 2001 slump.

The empirical plausibility of a link between monetary policy and macroeconomic instability was first established by Clarida, Gali and Gertler (2000). They estimate variants of the Taylor rule and their research suggests that the Federal Reserve's policy may have steered the economy into an indeterminate equilibrium during the 1970s. Yet, they also find that the changes to policy which have taken place after 1980 – essentially a more aggressive response to inflation – brought about a stable and determinate environment. Lubik and Schorfheide (2004) reinforce this point but they refrain from using a single equation approach. They recognize that indeterminacy is a property of a system and apply Bayesian estimation techniques to a general equilibrium model. Their results parallel the earlier findings that the U.S. economy veered from an indeterminacy to a determinacy regime around 1980 – largely as the result of a more aggressive response of monetary policy towards inflation.

Moreover, this monetary policy change had perhaps an even greater influence on the economy: the transformation from the Great Inflation of the 1970s to the Great Moderation is often conjoined to the change in the ways monetary policy is conducted.[2] Yet, the Great Moderation came to an end sometime during the 2000s, and it was followed by enormous economic volatility.

Our aim is to examine the possible connection between this transformation and an alteration in the Federal Reserve's conduct of monetary policy. In particular, we concentrate on the effects of a possibly too easy monetary policy after the 2001

[2] See, for example, Benati and Surico (2009), Bernanke (2012), Coibion and Gorodnichenko (2011).

slump. We frame the analysis from the perspective of the (in-)determinacy debate and conduct it under the umbrella of the Bernanke versus Taylor dispute by considering three measures of inflation that repeatedly occur in the discussion: CPI, PCE as well as core PCE.

Accordingly, we estimate a small-scale New Keynesian model allowing for indeterminacy over the period between the 2001 slump and the onset of the Great Recession, thus, the NBER-dated 2002:I-2007:III window to be precise. To test for indeterminacy, we employ the method of Lubik and Schorfheide (2004).[3] This approach enables us to compute the probabilities of determinacy and indeterminacy regimes. We establish a number of new insights regarding U.S. central bank policy. For example, we can indeed expose a violation of the Taylor principle for most of the 2000s when using CPI inflation. This finding supports the visual inspection checks based on single equations in Taylor (2012) who coined the phrase *Great Deviation* to refer to this period. Hence, the 2002:I to 2007:III period would appear to be best described by an indeterminate version of the New Keynesian model. However, our upshot is different when basing the analysis on PCE data: we can neither rule in nor rule out indeterminacy. Finally, the evidence in favor of indeterminacy altogether vanishes when we use core PCE. Monetary policy then appears to have been quite appropriate. This conclusion parallels the insight from Bernanke's (2015) counterfactual Federal Funds rate.

The conflicting indeterminacy results that we obtain with the respective inflation indicators lead us to consider whether our results are an artifact of the six year sample of data. In particular, one can reasonably question the extent to which our results are driven by the priors as opposed to the data. To address this issue, we re-estimate the model on rolling windows of fixed length (23 quarters to match the length of the 2002:I-2007:III period) focussing on CPI inflation. The outcomes of the indeterminacy test performed on rolling windows are plausible. In particular, we identify only two broad periods (i.e. several consecutive windows) in which a passive policy has likely led to indeterminacy: the 1970s and the post-2001 period. The first period, which

[3] See Hirose (2014) and Ascari and Bonomolo (2015) for recent applications and Farmer, Khramov and Nicolo (2015) for an easily implementable procedure.

coincides with the span of the Burns and Miller Chairmanships, exactly matches the indeterminacy duration and the timing of a switch to determinacy in 1980 that Coibion and Gorodnichenko (2011) document. We take this analogy as a reassuring validation of our short window size approach, i.e. even though our period of interest is quite short, it is possible to infer meaningful information from it.[4]

Finally, we attend the issue of how best to measure inflation in the New Keynesian model. We address the ambiguity between the theoretical concept and the empirical inflation proxies by employing the methodology proposed by Boivin and Giannoni (2006) to exploit the relevant information from a data-rich environment. Accordingly, we combine all three measures of inflation in the measurement equation and re-estimate our model. CPI and PCE emerge as better indicators of the concept of inflation than core PCE. Moreover, indeterminacy cannot be ruled out.

Perhaps most closely related to our work are Belongia and Ireland (2015) and Jung and Katayama (2014) who, like us, evaluate the Federal Reserve's monetary policy during the 2000s.[5] In particular, Belongia and Ireland (2015) estimate a time-varying VAR to track the evolution of Federal Reserve policy that occurred through the 2000s. They find evidence of a change in the Federal Reserve's behavior away from stabilizing inflation towards stabilizing output and also of persistent deviations from the estimated policy rule. While similar in spirit to our results they and the other mentioned papers cannot address issues of indeterminacy.

The remainder of the paper evolves as follows. The next section sketches the model and its solution. Section 3 presents the econometric strategy and baseline results. Robustness checks are conducted in Section 4. Section 5 relaxes the assumption that model inflation is properly measured by a single empirical indicator. Section 6 concludes.

[4] Judd and Rudebusch (1998) is another example of an evaluation of monetary policy over a similarly short sample period.

[5] See Fackler and McMillin (2015), Fitwi, Hein and Mercer (2015) and Groshenny (2013) for related exercises.

2 Model

The familiar three linearized equations summarize our basic New Keynesian model:

$$y_t = E_t y_{t+1} - \tau(R_t - E_t \pi_{t+1}) + g_t \qquad \tau > 0 \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(y_t - z_t) \qquad \kappa > 0,\ 0 < \beta < 1 \tag{2}$$

and

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)(\psi_\pi \pi_t + \psi_y [y_t - z_t]) + \epsilon_{R,t} \qquad 0 \leq \rho_R < 1 \tag{3}$$

Here y_t stands for output, R_t denotes the interest rate and π_t symbolizes inflation. E_t represents the expectations operator. Equation (1) is the dynamic IS relation reflecting an Euler equation. Equation (2) describes the expectational Phillips curve. Finally, equation (3) represents monetary policy, i.e. a Taylor-type nominal interest rate rule in which $\psi_\pi > 0$ and $\psi_y > 0$ are chosen by the central bank and echo its responsiveness to inflation and the output gap. The term $\epsilon_{R,t}$ denotes an exogenous monetary policy shock whose standard deviation is given by σ_R. Fundamental disturbances involve exogenous shifts of the Euler equation which are captured by the process g_t as well as shifts of the marginal costs of production captured by z_t. Both variables follow AR(1) processes:

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t} \qquad 0 < \rho_g < 1 \tag{4}$$

and

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t} \qquad 0 < \rho_z < 1. \tag{5}$$

$\epsilon_{R,t}$, $\epsilon_{g,t}$ and $\epsilon_{z,t}$ are $i.i.d.\mathsf{N}(0, \sigma_\epsilon^2)$. Finally, the term $\rho_{g,z}$ denotes correlation between the demand and supply innovations. In sum, the vector of model parameters entails

$$\theta \equiv \left[\psi_\pi, \psi_y, \rho_R, \beta, \kappa, \tau, \rho_g, \rho_z, \rho_{g,z}, \sigma_R, \sigma_g, \sigma_z\right]'.$$

Indeterminacy implies that fluctuations in economic activity can be driven by arbitrary, self-fulfilling changes in people's expectations (i.e. sunspots). Concretely, in our simple New Keynesian model indeterminacy occurs when the central bank passively responds to inflation changes, i.e. when $\phi_\pi < 1 - \phi_y (1 - \beta) / \kappa$.

We follow the solution method proposed by Lubik and Schorfheide (2003). The full set of rational expectations solutions takes the form to

$$s_t = \Phi(\theta)s_{t-1} + \Phi_\varepsilon(\theta, \widetilde{\mathbf{M}})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t \tag{6}$$

where s_t is a vector of model variables,

$$s_t \equiv [y_t, R_t, \pi_t, E_t y_{t+1}, E_t \pi_{t+1}, g_t, z_t]'$$

ε_t is a vector of fundamental shocks and ζ_t is a non-fundamental sunspot shock.[6] The coefficient matrices $\Phi(\theta)$, $\Phi_\varepsilon(\theta, \widetilde{\mathbf{M}})$ and $\Phi_\zeta(\theta)$ are related to the structural parameters of the model. The sunspot shock satisfies $\zeta_t \sim i.i.d.\mathsf{N}(0, \sigma_\zeta^2)$. Indeterminacy can manifest itself in two ways: (i) through pure extrinsic non-fundamental shocks, ζ_t (a. k. a sunspots), disturbing the economy and (ii) it may affect the propagation mechanism of fundamental shocks through $\widetilde{\mathbf{M}}$.

3 Estimation and Baseline Results

This section describes the data as well as the estimation strategy. It is followed by a presentation and discussion of our baseline results.

3.1 Data and priors

Following Lubik and Schorfheide (2004) we replace $\widetilde{\mathbf{M}}$ in equation (6) with $\mathbf{M}^*(\theta) + \mathbf{M}$ where $\mathbf{M} = [M_{R\zeta}, M_{g\zeta}, M_{z\zeta}]'$. We select $\mathbf{M}^*(\theta)$ such that the responses of the endogenous variables to fundamental shocks are continuous at the boundary between the determinacy and the indeterminacy regions and the prior mean for $\mathbf{M}$ equals zero.

We use HP-filtered real per capita GDP and the Federal Funds Rate as our observable for output and the nominal interest rate. These choices make our empirical analysis comparable to Lubik and Schorfheide (2004). To draw up our analysis in the Bernanke versus Taylor debate, we consider in turn three different measures of inflation: CPI, PCE deflator and core PCE (annualized percentage changes). The data covers the period between the 2001 slump and the onset of the Great Recession,

[6]Under determinacy, the solution (6) boils down to $s_t = \Phi^D(\theta)s_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$.

i.e. 2002:I to 2007:III. Our baseline priors are identical to the ones in Lubik and Schorfheide (2004) and they are reported in Table 1.

3.2 Testing for indeterminacy

For each measure of inflation, we estimate the model over the two different regions of the parameter space, i.e. determinacy and indeterminacy. To assess the quality of the model's fit to the data we present data densities and posterior model probabilities for both parametric zones. We approximate the data densities using Geweke's (1999) modified harmonic mean estimator. Table 2 reports our results for each measure of inflation.

Following Taylor (2007, 2012), we begin by using headline CPI to measure inflation. In this case, the data favors the indeterminate model: the posterior probability of indeterminacy is around 0.90. This result suggests that Taylor's characterization of the Federal Reserve's monetary policy as *too low for too long* is in fact consistent with indeterminacy and potentially has veered the economy into instability.

Yet, the upshot differs depending on which measure of inflation we employ in the estimation. Take Bernanke's (2015) suggestion that Taylor's counterfactual experiment should have been performed with core PCE. When making this choice, the posterior probability for our sample concentrates all of its mass in the determinacy region. This result flags that the Federal Reserve had not been responding passively to inflation during this period.

However, the Humphrey-Hawkins reports to Congress document that the Federal Reserve based monetary policy deliberations on headline PCE from the beginning of 2000 until mid-2004. Since Taylor is particularly critical of the monetary policy from 2002 to 2004, we next measure inflation using headline PCE data. We repeat the estimation and the finding is now ambiguous: the probability of determinacy is 0.58. Phrased alternatively, we cannot dismiss the possibility of indeterminacy.

In sum, we find that indeterminacy outcomes are dependent on the measure of inflation that is used. In fact, this lines up with the Taylor and Bernanke debate. Before delving into the question of which measures are more appropriate, we will present more details on the estimation results.

3.3 Posterior estimates

Table 3 reports the posterior estimates of the model parameters. The table includes the respectively favored models for CPI and core PCE inflation.[7] The estimated policy rule's response to inflation, ψ_π, which essentially generates the indeterminacy, differs significantly depending on the way we measure inflation. In particular, when basing the estimation on CPI, the posterior mean equals 0.84 (with 90-percent interval $[0.61, 0.98]$). This result indicates that monetary policy violated the Taylor principle over the 2002-2007 period or in the words of Taylor:

> "[t]he responsiveness appears to be at least as low as in the late 1960s and 1970s." [Taylor, 2007, 469]

The opposite result ensues when using core PCE. In that case, the posterior mean of ψ_π is well above one at 3.01 (with 90-percent interval $[1.97, 4.17]$). In some sense, our findings highlight the source of the controversy between Taylor and Bernanke: the respective interpretations are closely related to the employed inflation measures.

Table 3 also shows the posterior estimates of $\mathbf{M}$ under indeterminacy. The elements of $\mathbf{M}$ are substantially different from zero which explains why indeterminacy materially affects the propagation of fundamental shocks which we will discuss next.

3.4 Propagation dynamics

We now turn briefly to a comparison of the propagations of fundamental shocks. In particular, Figure 1 depicts the impulse responses of output, inflation and the nominal interest rate under indeterminacy (the model being estimated using CPI inflation) while Figure 2 graphs the responses under determinacy (using core PCE inflation). Solid lines track the posterior means while the shaded areas cover the 90-percent probability intervals. The first and second rows of the figures show the responses to monetary policy as well as cost-push shocks. For these two disturbances, the patterns of the key model variables look remarkably similar across the indeterminate and the

[7] The appendix reports results for parameter estimates, variance decomposition and impulse responses when using headline PCE inflation data conditional on both determinacy and indeterminacy.

determinate versions of the model. This reaction contrasts with the responses to aggregate demand shocks (plotted third rows of Figures 1 and 2). While on impact output increases in both regimes, the responses of inflation are starkly opposite. The determinate model's responses to an aggregate demand shock are conventional: inflation increases and the central bank tightens its policy by raising the nominal interest rate.

On the other hand, the transmission of aggregate demand disturbances changes qualitatively in the indeterminate model (favored when using CPI in the estimation). Now, output increases but inflation moves in the opposite direction. This response appears to reflect the alternative propagation of fundamental shocks in model versions that feature indeterminacy. These propagation dynamics are captured by the elements of the matrix **M**. In particular, the posterior estimate of $M_{g\zeta}$ is far from zero at -1.99 and as such qualitatively alters the dynamics of a demand shock. This result can be understood as follows. Given that the economy sits in the indeterminacy zone, monetary policy is passive and both inflation and the nominal interest rate fall. The net effect is a drop of the expected real return and increases current output. In some way, the Federal Reserve – by creating indeterminacy – transformed demand shocks to look qualitatively similar to supply shocks.

Let us also discuss the model's reaction to sunspots. The fourth row of Figure 1 displays the dynamics that arise if the economy is hit by an inflationary sunspot shock. The impulse responses show that the disturbance reduces the expected real return which subsequently increases current consumption and, hence, output. The Phillips curve then translates this effect into a rise of inflation thereby creating a self-fulfilling cycle: higher inflation expectations leading to higher actual inflation.

3.5 Variance decomposition

The unconditional forecast error variance decomposition at the posterior mean for output (deviations from trend), inflation and interest rates are reported in Table 4. Following Lubik and Schorfheide (2004), we orthogonalize the ε_{gt} and ε_{zt} shocks such that the cost-push shock only affects ε_{zt} and the demand shock affects both ε_{gt} and ε_{zt}.

Two main messages arise from the variance decomposition. Firstly, the economy's regimes imply different shocks as the prime driver of fluctuations: in the indeterminacy regime, cost-push shocks cause over 80 percent of output fluctuations whereas in the determinacy case aggregate demand disturbances play the main role. Secondly, sunspot shocks' importance is only marginal with the most significant contribution being eight percent in explaining the variance decomposition of the policy rate. To conclude this section, the choice of the inflation measure implies not only different results regarding the likeliness of determinacy, the choice also entails contrasting interpretations of the causes of macroeconomic fluctuations.

4 Sensitivity analysis

This section will test the sensitivity of our results in various directions. The robustness checks involve (i) testing for indeterminacy on rolling windows, (ii) alternative priors for key parameters (ψ_π and π^*), (iii) alternative measure of the output gap and (iv) model extensions.

Rolling windows To address the issue of whether the results are an artifact of our small sample size, we re-estimate the model on rolling windows of fixed length. More precisely, we keep the size of the windows at 23 quarters to match the number of observations in our period of interest. We start estimating the model using data from the first window (1966:I-1971:III) and then repeat the estimation by moving the window quarter by quarter.[8] Here we just use CPI to measure inflation since it is only in the 2000s that the Federal Reserve began to base its monetary policy deliberations on PCE and core PCE. Figure 3 presents the evolution of the posterior probability of determinacy for the U.S. economy from 1966:I to 2008:III. The end point is chosen to avoid obvious complications that emanate from hitting the lower interest rate bound. The graph suggests that the U.S. economy was likely in a state of

[8] This approach to estimate linear DSGE models was recently promoted by Canova (2009), Canova and Ferroni (2011a) and Castelnuovo (2012). Rolling window estimation provides two benefits. It allows us to uncover time-varying patterns of the model's parameters, in particular, of the monetary policy coefficients. At the same time, the procedure permits us to remain within the realm of linear models and apply standard Bayesian methods.

indeterminacy during the 1970s. Thereafter, beginning with the Volcker disinflation policies, the economy shifted back to a determinate equilibrium which lasted until the end of the 2001 recession. These findings are consistent with related studies such as Clarida, Gali and Gertler (1999), Lubik and Schorfheide (2004) and Coibion and Gorodnichenko (2011).[9] We take this correspondence as a justification for estimating our model based on a short window.

Alternative priors One possible drawback to using a small sample size is that the prior might *speak louder* than the data. To make our empirical analysis transparent, the priors we employ in our baseline estimation (Table 1) were set identical to the ones used by Lubik and Schorfheide (2004). Accordingly, our baseline specification implies a prior probability of determinacy equal to 0.53. As a robustness check, we tilt the prior probability mass toward the determinacy region. Specifically, we change the prior mean of ψ_π from 1.1 to 1.3 and in doing so we ramp up the prior probability of determinacy from 0.53 to 0.7. Thus, the Lubik and Schorfheide test finds it harder to favor indeterminacy. Table 5 reports the posterior probabilities of (in-)determinacy under this alternative prior for each measure of inflation. The results remain largely unaltered. For example, the odds of indeterminacy versus determinacy are still five to one when estimating the model using CPI inflation. This finding provides some further support for our results.

So far, the prior mean for π^* was set at four percent. One may question our results on the grounds that this number seems too high for the analysis of the 2000s given the Federal Reserve's implicit inflation target was closer to two percent. As a further robustness check we return the prior mean of ψ_π back to 1.1 while now reducing the prior mean of π^* to two. Again, Table 5 reports that our results remain unchanged.

Alternative measure of the output gap The next check for robustness involves measuring the output gap based on the Congressional Budget Office's estimate of potential output as in Belongia and Ireland (2015) and others. Table 5 suggests that,

[9] Figure 3 is comparable to Coibion and Gorodnichenko (2011, Figure 4) who plot the probability of determinacy implied by the distribution of time-varying parameters. Coibion and Gorodnichenko report a moving average which makes their series smoother than ours.

again, our results remain robust.

Model extension It is well known that the determinate New Keynesian model features a poor internal propagation mechanism while potentially exhibiting richer dynamics under indeterminacy. Accordingly, the posterior mass might be biased toward the indeterminacy region. Hence, following Lubik and Schorfheide (2004), we extend the model with consumption habits. Table 6 points again to robustness. The relative posterior probabilities between the baseline model under indeterminacy and the habit model under determinacy carry over from the benchmark exercise (Table 2).[10]

5 Which measure of inflation to chose?

So far, our estimations have delivered mixed evidence regarding the probability of indeterminacy for the 2002:I to 2007:III period. Our results are consistently dependent on the specific measures of inflation which have been chosen in the estimation – only if the estimation uses the core PCE series can we comfortably rule out indeterminacy. Thus, while the assumption that theoretical concepts are observed by the econometrician is made in the estimation of DSGE models, it may be the case that the data only provides an imperfect indicator of the model concept. Thus, it is plausible to think that all three or only a subset of the measures of inflation contain relevant information.

One can view this idea in at least two ways. Firstly, a single series potentially contains a small amount of information relative to the information that is available and used by agents. Secondly, different inflation series may be useful for distinctive parts of the model. In this line of thinking, we will depart from the assumption that model inflation is measured by a single series and draw on Boivin and Giannoni's (2006) *data-rich environment* application of dynamic factor analysis.[11] In a nutshell,

[10]With habits, the model does not have an exploitable analytical boundary condition along which it crosses from the determinacy to indeterminacy zones. Without this, we are not able to estimate the **M** vector and therefore not able to estimate the indeterminate model in a consistent fashion with the approach taken in the previous sections.

[11]The approach builds on Sargent (1989) and Forni, Hallin, Lippi and Reichlin (2000). Canova

we want to exploit the relevant information from various inflation series in the estimation to deliver more robust results. We treat the model concept of inflation as the unobservable common factor for which data series are imperfect proxies.

More concretely, the estimation involves the transition equation (6)

$$s_t = \Phi(\theta)s_{t-1} + \Phi_\varepsilon(\theta, \widetilde{M})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t$$

or its determinacy equivalent

$$s_t = \Phi^D(\theta)s_{t-1} + \Phi^D_\varepsilon(\theta)\varepsilon_t \tag{7}$$

and the measurement equation

$$\begin{bmatrix} GDP_t \\ FFR_t \\ \mathbf{X}_t \end{bmatrix} = \begin{bmatrix} 0 \\ r^* + \pi^* \\ \underset{3\times1}{\mathbf{0}} \end{bmatrix} + \begin{bmatrix} \mathbf{I}_2 & \underset{2\times3}{\mathbf{0}} \\ \underset{3\times2}{\mathbf{0}} & \mathbf{\Lambda} \end{bmatrix} \begin{bmatrix} y_t \\ 4R_t \\ \boldsymbol{\pi}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \mathbf{u}_t \end{bmatrix}. \tag{8}$$

Here GDP_t stands for HP-filtered per capita GDP, FFR_t denotes the Federal Funds rate, $\mathbf{X}_t \equiv [CPI_t, PCE_t, corePCE_t]'$ is the vector of empirical proxies of inflation, $\mathbf{\Lambda}$ =diag$(\lambda_{CPI}, \lambda_{PCE}, \lambda_{corePCE})$ is a 3×3 diagonal matrix of factor loadings relating the latent model concept of inflation to the three empirical proxies, $\boldsymbol{\pi}_t \equiv 4[\pi_t, \pi_t, \pi_t]'$ and $\mathbf{u}_t = [u_t^{CPI}, u_t^{PCE}, u_t^{corePCE}]' \sim i.i.d.(\mathbf{0}, \mathbf{\Sigma})$ is a vector of serially and mutually uncorrelated proxy specific measurement errors with $\mathbf{\Sigma}$ =diag$(\sigma^2_{CPI}, \sigma^2_{PCE}, \sigma^2_{corePCE})$. Lastly, y_t, R_t and π_t are the model's state variables.

We jointly estimate the parameters $(\mathbf{\Lambda}, \mathbf{\Sigma})$ of the measurement equation (8) along with the structural parameters θ since (6)-(8) represent a state space system. We standardize all three measures of inflation by calibrating π^* equal to 2.5 percent – which stands for an average of the three inflation series – as there is only one model concept of π^*. The standardization permits us to interpret the factor loadings, λ_j with $j \in \{CPI, PCE, corePCE\}$, as correlations between the latent model concept of inflation and the respective observables.[12] Our prior distribution for the loadings and measurement errors are $\lambda_j \sim Beta(0.50, 0.25)$ and $u_t^j \sim \mathsf{N}(0.10, 0.20)$ respectively.[13]

and Ferroni (2011) and Castelnuovo (2013) are recent applications.

[12] See Forni, Hallin, Lippi and Reichlin (2000).

[13] The figures reported in brackets refer to the mean and standard deviation of the distributions.

By employing a beta distribution, the support of the λ_j is restricted to the open interval $(0,1)$ which is a necessary sign restriction. Note that the identification of the parameters in the measurement equation is obtained under the conditions stated in Geweke and Zhou (1996, Section 3).

Table 7 reports the resulting log-data densities which are -133.24 for determinacy and -132.54 for indeterminacy. Phrased differently, the posterior probabilities are 33% versus 67% respectively. While the evidence is mixed at best, the result indicates that we cannot strictly rule out indeterminacy.

As mentioned before, one can think of our setup as a factor model where the model concept of inflation is defined as the common factor to the imperfect indicators produced by our three different measures. This signal extraction setup offers the advantage that the standardized measures of inflation employed in the estimation allows us to interpret the factor loadings as correlations between the latent factor and the respective observables. Table 8 reports the posterior estimates of the model parameters along with the loadings on inflation and the standard deviation of the associated measurement errors. Conditional on both determinacy and indeterminacy the factor loadings on CPI and PCE are roughly three times as high as the loading on core PCE. These numbers imply that CPI and PCE inflation measures contribute relatively more toward the meaning of the latent model concept of inflation while core PCE is relatively less important. Furthermore, there is evidence of substantial indicator-specific component for the core PCE indicator of inflation as evident in its high standard deviation of measurement error relative to that of the other two measures. In sum, (i) CPI and PCE provide better indicators of the latent concept of inflation while core PCE, despite being promoted by Bernanke (2015), is less informative; (ii) when we combine all three measures of inflation in our estimation, we find that indeterminacy cannot be ruled out.

6 Concluding remarks

The Taylor rule has become a benchmark for evaluating the Federal Reserve's policy. Along these lines, this paper estimates a New Keynesian model of the U.S. economy over the period following the 2001 slump, a period over which the conduct of monetary policy has been criticized by some commentators. Our assessment varies with the measure of inflation that is put into the model estimation. When measuring inflation with CPI, we find support for the view that monetary policy during these years was extra easy and led to equilibrium indeterminacy. Instead, if the estimation involves core PCE, monetary policy comes out as active and the evidence for indeterminacy dissipates. Our take on this result is that each of the inflation series is only an imperfect proxy for the artificial economy's concept of inflation. We re-formulate the artificial economy as a factor model where the theory's concept of inflation is the common factor to the alternative empirical inflation series. Again, extra easy monetary policy as well as indeterminacy cannot be ruled out. In sum, while not completely resolving the ongoing debate between Bernanke, Taylor and others, our study sheds further light on the effects of U.S. monetary policy during the years leading up to the Great Recession.

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Table 1 - Prior Distribution for DSGE Model Parameters

Name	Range	Density	Mean	Standard deviation	90-percent interval
ψ_π	$\mathbb{R}^+$	Gamma	1.10	0.50	[0.33,1.85]
ψ_y	$\mathbb{R}^+$	Gamma	0.25	0.15	[0.06,0.43]
ρ_R	[0,1)	Beta	0.50	0.20	[0.18,0.83]
π^*	$\mathbb{R}^+$	Gamma	4.00	2.00	[0.90,6.91]
r^*	$\mathbb{R}^+$	Gamma	2.00	1.00	[0.49,3.47]
κ	$\mathbb{R}^+$	Gamma	0.50	0.20	[0.18,0.81]
τ^{-1}	$\mathbb{R}^+$	Gamma	2.00	0.50	[1.16,2.77]
ρ_g	[0,1)	Beta	0.70	0.10	[0.54,0.86]
ρ_z	[0,1)	Beta	0.70	0.10	[0.54,0.86]
ρ_{gz}	[-1,1]	Normal	0.00	0.40	[-0.65,0.65]
$M_{R\zeta}$	$\mathbb{R}$	Normal	0.00	1.00	[-1.64,1.64]
$M_{g\zeta}$	$\mathbb{R}$	Normal	0.00	1.00	[-1.64,1.64]
$M_{z\zeta}$	$\mathbb{R}$	Normal	0.00	1.00	[-1.64,1.64]
σ_R	$\mathbb{R}^+$	Inverse Gamma	0.31	0.16	[0.13,0.50]
σ_g	$\mathbb{R}^+$	Inverse Gamma	0.38	0.20	[0.16,0.60]
σ_z	$\mathbb{R}^+$	Inverse Gamma	1	0.52	[0.42,1.57]
σ_ζ	$\mathbb{R}^+$	Inverse Gamma	0.25	0.13	[0.11,0.40]

Notes: The inverse gamma priors are of the form $p\left(\sigma|\upsilon,s\right)\propto\sigma^{-\upsilon-1}e^{-\frac{\upsilon s^2}{2\sigma^2}}$ where $\nu=4$ and s equals $0.25, 0.3, 0.6$ and 0.2, respectively. The prior for ρ_{gz} is truncated to ensure that the correlation lies between -1 and 1. The prior predictive probability is 0.527.

Table 2: Determinacy versus Indeterminacy - 2002:I-2007:III

	Log-data density		Probability	
Inflation measure	Determinacy	Indeterminacy	Determinacy	Indeterminacy
CPI	-95.48	-93.28	0.10	0.90
PCE	-85.42	-85.75	0.58	0.42
Core PCE	-64.60	-71.58	1	0

Notes: According to the prior distributions, the probability of determinacy is 0.527.

Table 3 - Parameter Estimation Results

	CPI		Core PCE	
	Mean	90-percent interval	Mean	90-percent interval
ψ_π	0.84	[0.61, 0.98]	3.01	[1.97,4.17]
ψ_y	0.19	[0.05, 0.41]	0.28	[0.07,0.64]
ρ_R	0.83	[0.74, 0.90]	0.76	[0.64,0.85]
π^*	3.28	[1.27, 6.01]	1.99	[1.67,2.31]
r^*	1.15	[0.47, 2.01]	1.40	[0.84,2.01]
κ	0.91	[0.51, 1.41]	0.71	[0.31,1.19]
τ^{-1}	1.66	[1.00, 2.49]	1.62	[0.95,2.48]
ρ_g	0.60	[0.45, 0.73]	0.80	[0.72,0.87]
ρ_z	0.80	[0.68, 0.89]	0.61	[0.49,0.74]
ρ_{gz}	-0.28	[-0.72, 0.17]	0.86	[0.57,0.97]
$M_{R\zeta}$	-0.57	[-1.90, 1.00]		
$M_{g\zeta}$	-1.99	[-2.92, -1.05]		
$M_{z\zeta}$	0.41	[0.05, 0.83]		
σ_R	0.16	[0.12, 0.21]	0.16	[0.12,0.21]
σ_g	0.28	[0.18, 0.40]	0.19	[0.14,0.25]
σ_z	0.74	[0.54, 1.03]	0.62	[0.47,0.82]
σ_ζ	0.20	[0.12, 0.30]		

Notes: The table reports posterior means and 90-percent probability intervals of the model parameters. CPI posteriors are conditional on indeterminacy. Core PCE posteriors are conditional on determinacy. Under determinacy, the $M's$ and σ_ζ disappear. Hence, the entries are left blank. The posterior summary statistics are calculated from the output of the Metropolis Hastings algorithm.

Table 4: Variance Decomposition

	Variables\Shocks	ε_R	ε_g	ε_z	ζ
CPI (Indet.)	y	9.44	7.47	82.37	0.71
	π	21.82	54.53	16.45	7.20
	R	1.29	74.28	16.24	8.20
Core PCE (Det.)	y	1.99	83.57	14.43	-
	π	39.25	31.03	29.72	-
	R	7.51	69.37	23.12	-

Variance decompositions are performed at the mean of the posterior distribution of the model's parameters.

Table 5: Determinacy versus Indeterminacy - 2002:I-2007:III (Robustness)

		Log-data density		Probability	
Inflation measure		Det.	Indet.	Det.	Indet.
CPI	Alternative prior for ψ_π	-95.04	-93.58	0.19	0.81
	Alternative prior for π^*	-95.63	-93.29	0.09	0.91
	CBO output gap	-97.89	-95.85	0.12	0.88
PCE	Alternative prior for ψ_π	-85.04	-85.98	0.72	0.28
	Alternative prior for π^*	-85.51	-85.73	0.55	0.45
	CBO output gap	-88.08	-88.18	0.53	0.47
Core PCE	Alternative prior for ψ_π	-64.47	-71.74	1	0
	Alternative prior for π^*	-64.71	-71.01	1	0
	CBO output gap	-68.53	-73.63	0.99	0.01

Notes: The alternative prior for ψ_π implies setting the prior mean to 1.295 which increases the prior probability of determinacy to 0.7. The alternative prior for π^* implies setting the prior mean to 2 which leaves the prior probability of determinacy unaltered.

Table 6: Determinacy versus Indeterminacy - 2002:I-2007:III (Robustness)

		Log-data density		
Inflation measure	Specification	Det.	Indet.	Probability
CPI	Benchmark	-95.48	-93.28	0.87
	Habit	-95.18		0.13
PCE	Benchmark	-85.42	-85.75	0.26
	Habit	-84.70		0.74
Core PCE	Benchmark	-64.60	-71.58	0
	Habit	-62.73		1

Table 7: Determinacy versus Indeterminacy - 2002:I-2007:III (DSGE factor model)

Log-data density		Probability	
Determinacy	Indeterminacy	Determinacy	Indeterminacy
-133.24	-132.54	0.33	0.67

Notes: According to the prior distributions, the probability of determinacy is 0.527.

Table 8 - Parameter Estimation Results (DSGE factor model)

	Determinacy		Indeterminacy	
	Mean	90-percent interval	Mean	90-percent interval
ψ_π	2.05	[1.25,3.03]	0.81	[0.56,0.97]
ψ_y	0.27	[0.07,0.59]	0.19	[0.05,0.40]
ρ_R	0.85	[0.78,0.91]	0.83	[0.74,0.90]
r^*	1.09	[0.50,1.82]	1.32	[0.59,2.20]
κ	0.76	[0.40,1.23]	0.93	[0.50,1.44]
τ^{-1}	1.83	[1.10,2.71]	1.63	[0.98,2.46]
ρ_g	0.79	[0.71,0.86]	0.60	[0.44,0.73]
ρ_z	0.62	[0.46,0.79]	0.79	[0.66,0.89]
ρ_{gz}	0.56	[0.09,0.90]	-0.26	[-0.69,0.19]
$M_{R\zeta}$			-0.54	[-1.95,1.06]
$M_{g\zeta}$			-2.07	[-2.98,-1.19]
$M_{z\zeta}$			0.37	[0.02,0.76]
σ_R	0.16	[0.12,0.21]	0.16	[0.12,0.21]
σ_g	0.19	[0.14,0.26]	0.28	[0.18,0.43]
σ_z	0.72	[0.52,0.99]	0.77	[0.55,1.08]
σ_ζ			0.20	[0.13,0.31]
λ_{CPI}	0.73	[0.50,0.92]	0.53	[0.33,0.78]
λ_{PCE}	0.75	[0.52,0.94]	0.55	[0.34,0.81]
$\lambda_{CorePCE}$	0.26	[0.06,0.50]	0.19	[0.05,0.40]
σ_{CPI}	0.30	[0.16,0.43]	0.31	[0.18,0.42]
σ_{PCE}	0.19	[0.11,0.34]	0.18	[0.10,0.32]
$\sigma_{CorePCE}$	0.92	[0.73,1.16]	0.91	[0.72,1.15]

Notes: The table reports posterior means and 90-percent probability intervals of the model parameters.

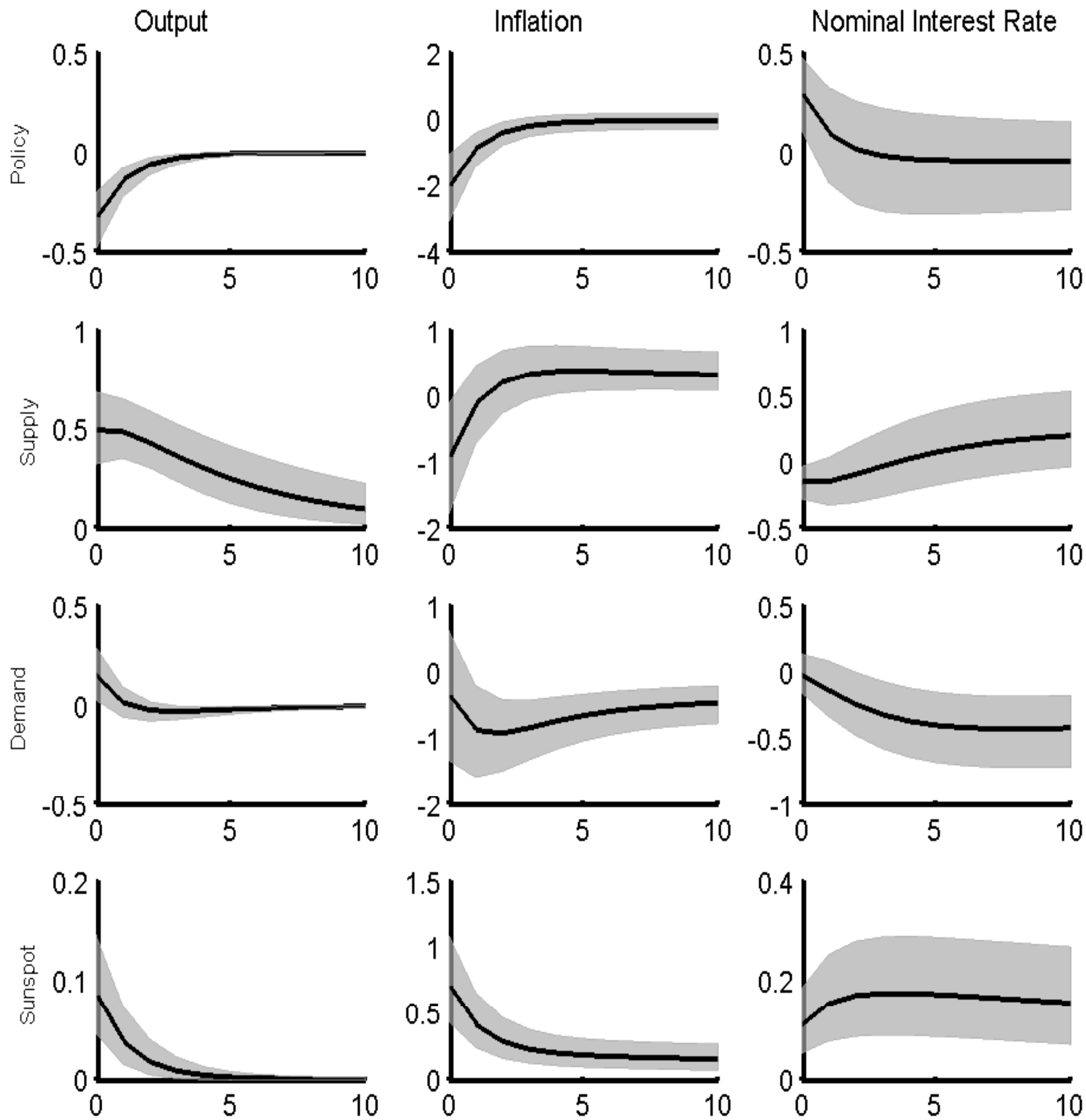

Figure 1: Impulse responses to one-standard-deviation shocks under indeterminacy from the model estimated over the period 2002:I - 2007:III using CPI inflation. Solid lines depict the posterior means and the shaded areas represent the 90-percent probability intervals.

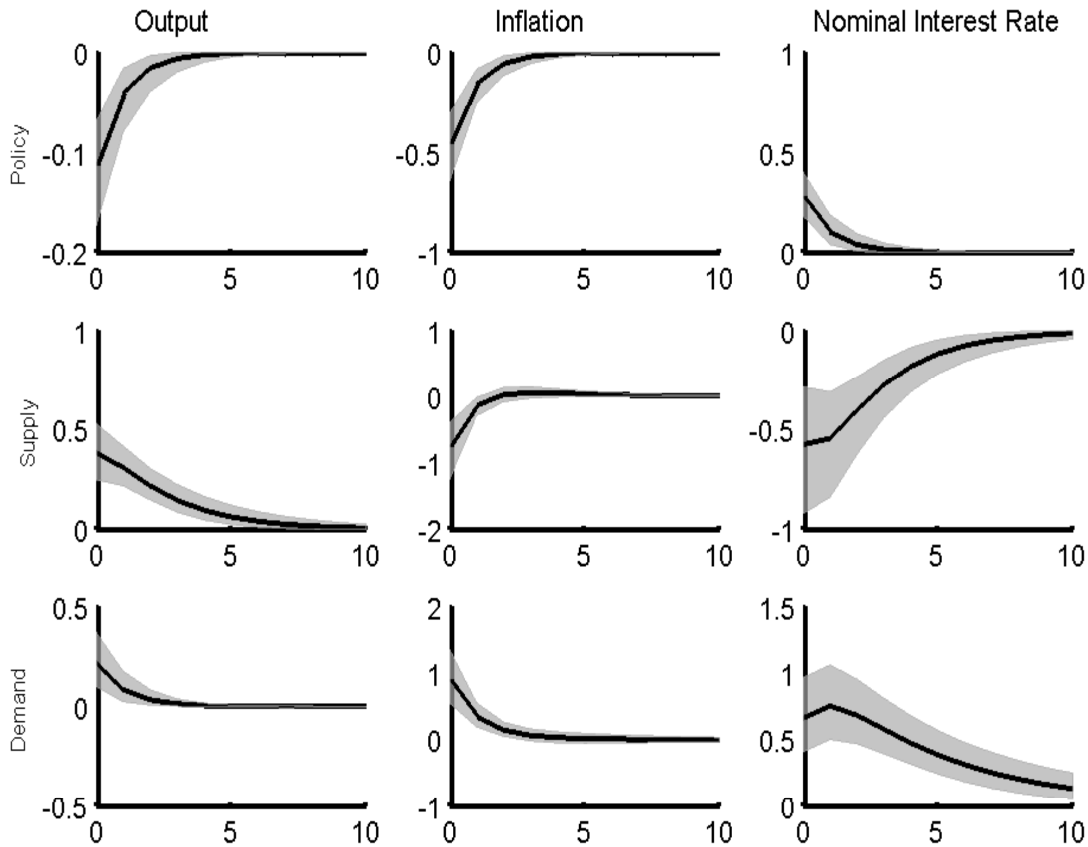

Figure 2: Impulse responses to one-standard-deviations shocks under determinacy from the model estimated over the period 2002:I - 2007:III using core PCE inflation. Solid lines depict the posterior means and the shaded areas represent the 90-percent probability intervals.

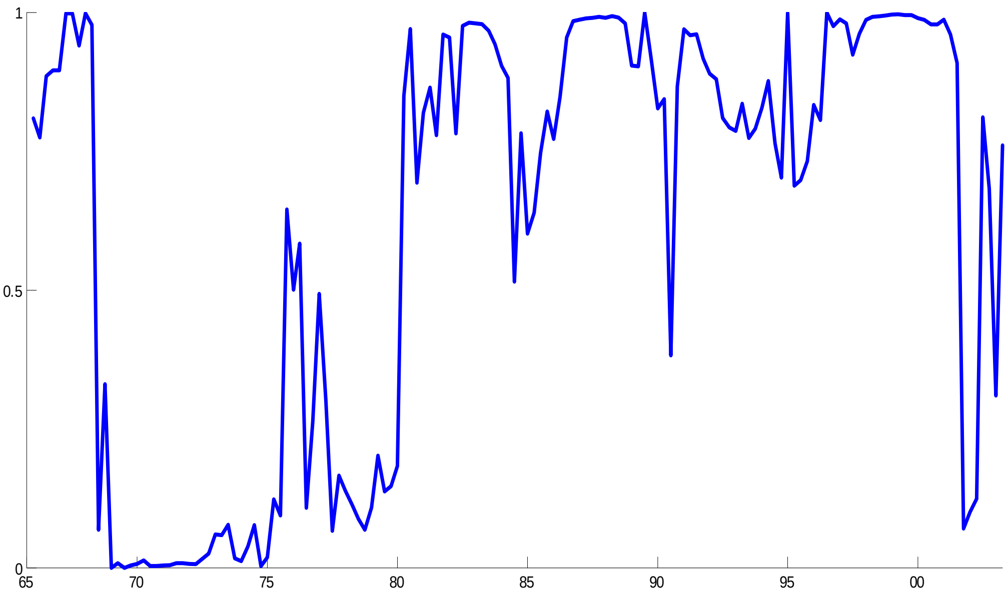

Figure 3: Probability of determinacy