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We develop a model of informational lobbying in which a policymaker must decide which issues to reform, but is uninformed about which issues he would be better off reforming. On each issue there is an informed interest group that always favors the adoption of reform, and which can lobby the policymaker by offering to provide verifiable information about the state of the world for its issue. A key feature of our model is that the policymaker faces time/resource constraints which may restrict both 1) his ability to grant access to and verify the information of all lobbying interest groups and 2) his ability to reform all issues. We show that when the policymaker can reform all issues, the act of lobbying by an interest group signals information only imperfectly. In particular, an interest group may want to lobby the policymaker even when it does not possess favorable information in the hope that the policymaker is unable to verify its information but still takes the act of lobbying as a signal that the state is favorable to reform. We call such lobbying behavior 'overlobbying'. We then show that a restriction on the number of issues on which reforms can be implemented, viz. an agenda constraint, can improve information transmission by eliminating overlobbying. Importantly, we identify circumstances in which an agenda constraint improves the ex ante welfare of the policymaker and of each interest group, leading to a Pareto improvement.

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"HYPERACTIVITY is not a virtue in a legislature. Winston Churchill thought Parliament should meet for no more than five months a year. Texas enjoys relative freedom from red tape partly because its state legislature meets only every other year. If the European Parliament sat only once every two years, the continent's regulation-infested economy might well be healthier." [from the Economist (2014)]

1. INTRODUCTION

Lobbying of policymakers by special interest groups is pervasive under a diverse range of political systems around the world. Each year millions of dollars are spent on lobbying activities (for instance, in the US \$3.24 bn were spent in the year 2014).[1] The offices of lobbying firms are an integral fixture of the political power centres around the world, be it D.C., Brussels or Canberra. Critiques often attribute a politician's advocacy/opposition of a certain piece of legislation to his/her cozy relationship with one or the other lobby. It is therefore no surprise that the nature, extent and impact of lobbying are topics of popular debate as well as of extensive scrutiny among scholars.

While the popular imagination often paints lobbying as an attempt to seek rents, special favors or exemptions in exchange for money or other lures,[2] a recent political economy literature treats lobbying as an activity that, either directly or indirectly, transmits policy relevant information from special interest groups to the policymaker. For instance, the act of lobbying by an interest group can serve as a signal of the availability of favorable information, which in turn induces the politician to grant the interest group access so that the information may be obtained or verified. While the source of this information is biased, a rational policymaker can, after accounting for such bias, make on average a better informed decision in the general interest.[3]

This paper contributes to the literature on informational lobbying by developing a model of lobbying in which the policymaker (hereafter, PM) faces time/resource constraints in both the provision of access to interest groups and the formulation of policy. The PM is responsible for multiple issues on which he must decide whether to implement a reform or to keep the status quo. The PM's optimal policy depends on whether the state of the world on that issue is pro-reform or not. The PM is initially uninformed about the state of the world. On each issue there is an interest group (hereafter, IG) wanting its issue to be reformed irrespective of the state of the world. Each IG has verifiable information about the state of the world on the issue it is associated with and can lobby the PM at a cost. The PM must choose whether to grant a lobbying IG access (or study the information the IG provided when lobbying). If the PM grants an IG access, he learns the state of the world on the issue that the IG is associated with.

Thus, we construct a model in which there is potential for welfare-enhancing information transmission via lobbying and access. However, we incorporate a re-

[1] See http://www.statista.com/statistics/257337/total-lobbying-spending-in-the-us/ for details.

[2] See Tullock (1980), Hillman and Riley (1989) and Grossman and Helpman (1994) for a formal treatment.

[3] See, for instance, Austen-Smith and Wright (1992). Also, see Gawande, Maloney and Montes-Rojas (2009) and Belloc (2015) for empirical evidence on the influential role of informational lobbying on policy choices.

alistic aspect of the policymaking process, viz. time/resource constraints.[4] In particular, we assume that the PM has limited time and resources that must be rationed between multiple IGs. Also, we take into account the fact that in a time period (say, a legislative session) only a certain number of policies/legislations may be enacted. Specifically, in our baseline model we assume there are two issues but the PM can give access to only one IG. We then consider two possibilities: one, where the PM can enact reform on both issues; and two, where the PM can enact reform on only one issue. We will refer to the former case as one where there is *no agenda constraint*, and the latter as the case with an *agenda constraint*.[5] Our model makes several interesting points regarding the nature of informational lobbying in the presence/absence of constraints on access as well as agenda.

First, we characterize the equilibria of the model both with and without agenda constraint and study the extent to which lobbying can transmit policy relevant information to the PM. Intuitively, lobbying provides the PM information via two channels, direct and indirect. The direct channel operates via granting of access whereby the PM observes an IG's information on the state of the world for the issue the IG is associated with. The indirect channel operates via the signalling potential of lobbying. Since lobbying is costly and since there is a chance that the PM will grant the IG access and observe the IG's information, it is relatively more attractive for an IG to lobby when it has favorable information than when it has unfavorable information. Hence, the act of lobbying by an IG can serve as a signal that it has favorable information. The extent to which lobbying can thus serve as a signal of the true state of the world depends on two factors: one, the cost of lobbying; and two, the extent to which the PM is able, via access, to "discipline" the IGs to lobby truthfully, which depends on the agenda constraint. When the cost of lobbying is sufficiently high, or when the PM can credibly discipline the IGs (which happens in presence of an agenda constraint), we obtain a separating equilibrium wherein each IG lobbies if and only if it has favorable information. When this is the case, the PM gets fully informed. However, if the lobbying cost is low or the access strategies do not adequately act as a disciplining mechanism (which happens in the absence of an agenda constraint) then we get a semi-separating equilibrium. For instance, in the baseline model, we can get an overlobbying equilibrium wherein an IG sometimes lobbies even when it has unfavorable information. It does so hoping that it will not be granted access and the PM will still believe with a sufficiently high probability that the state of the world is pro-reform and, therefore, implement the reform with positive probability.[6]

Second, we show that in the presence of an access constraint, the imposition

[4] The existence of resource constraints in policymaking is self-evident. The existence of time constraints that lawmakers are exposed to, forcing them to prioritize issues and limiting the number of IGs they can grant access to and verify the information, are documented extensively in the literature. For example, Bauer, Dexter and De Sola Pool (1963; 405) writes: "The decisions most constantly on [a Congressman's] mind are not how to vote, but what to do with his time, how to allocate his resources, and where to put his energy." Hall (1996; 24) reports a legislative assistant saying: "He [the Congressman] had a conflict, but the point is that members always have conflicts. They have to be in two places at once, so they have to choose: Which issue is more important to me?" Likewise, Jones and Baumgartner (2005; 147) writes: "The agenda space is severely limited, and many issues can compete for the attention of policymakers."

[5] Further in the paper we consider the general case where there are I issues, access can be granted only to K IGs, and reform can be implemented only on N issues, where $1 \leq K \leq N \leq I$.

[6] This intuition generalizes to the model with more than two issues. Moreover, in that model, we can also get underlobbying equilibria wherein an IG sometimes does not lobby in spite of having favorable information.

of an agenda constraint can improve information transmission. Specifically, in our baseline model, there is a range of lobbying costs for which the unique equilibrium in the absence of agenda constraint involves overlobbying, whereas imposing the agenda constraint leads to a perfectly informative equilibrium. However, the presence of an agenda constraint also has its costs, since such constraint precludes the PM from reforming all issues. Hence the net effect of an agenda constraint is ambiguous. However, we identify a range of parameter values for which the imposition of an agenda constraint improves the ex ante welfare of the PM as well as the ex ante welfare of every IG, thereby leading to a Pareto improvement. Thus, our model provides a rationale for institutional features (such as a restriction on the sitting time of a legislature per year) aimed at reducing legislative activity. It further illustrates that a reduction in legislative activity can be Pareto improving, not because it reduces wasteful rent seeking and bureaucratic red tape, as suggested in the quote from the Economist (2014) cited above, but because it improves IGs' information transmission to lawmakers.

Third, we show that our results extend in a natural way to a model with multiple issues. In that model we show that given an access constraint, there always exists a level of agenda constraint for which the PM gets perfectly informed in equilibrium and such equilibrium can lead to Pareto improvement relative to the case without an agenda constraint.

Fourth, we extend our model to allow the PM to have subpoena power, which gives the PM the option to access the information of an IG irrespective of whether the IG chooses to lobby or not. Subpoena power is shown to have two effects: one, it allows the PM to obtain information which he could not otherwise obtain; two, it lowers the incentives of IGs to lobby when they have favorable information. We show that the latter effect weakly dominates the former, and hence the introduction of subpoena power does not improve overall information transmission and the PM's ex ante welfare.

Finally, our baseline model takes the existence of the IGs as given and assumes that the IGs are pro-reform. In a supplementary online appendix we provide the microfoundations of our model and show that the set-up we consider is indeed consistent with the equilibrium of a more general game with endogenous formation of IGs.

The remainder of the paper is organized as follows. Section 2 reviews the most relevant literature. Section 3 presents our baseline model. Section 4 provides a numerical example that illustrates our main results and the intuition underlying them. Section 5 provides the full analysis of our baseline model. Section 6 considers various extensions to our baseline model. Section 7 concludes. All proofs are in the appendix. A supplementary online appendix studies extensions to the model and provides details of the proofs and calculations underlying our examples omitted in the paper.

2. RELATED LITERATURE

The novelty of our paper is to study informational lobbying in the presence of both an access constraint and an agenda constraint. Neither of the papers mentioned below (and, to our knowledge, no other existing paper) considers these two types of constraints simultaneously. Some of these papers consider a policy choice on a single issue, meaning that they cannot consider the possibility of an agenda constraint. Others consider a policy choice on multiple issues, but either deny the PM the ability to grant access and/or verify IGs' information, or consider IGs' production of public information, which would be equivalent to removing any access constraint in our setting.

Our paper is related to an extensive literature on informational lobbying and persuasion. Potters and van Winden (1992) and Lohmann (1993, 1995) are early contributions that model IG influence as a signalling game, where IGs are endowed with non-verifiable information that they can convey, at least in part, to a PM via a combination of cheap talk and costly signalling. Austen-Smith and Wright (1992) and Rasmusen (1993) model IGs' decisions to produce and convey verifiable information as well as the PM's decision to verify, at a cost, the information conveyed by IGs. All these papers differ from ours in that the source of inefficiency they identify lies in the ability of IGs to deceive the PM by telling him lies. Deception is not allowed in our setting where IGs can withhold information (by not lobbying) but cannot tamper with it.[7] By contrast, the source of inefficiency that we identify lies in the agenda constraint and the access constraint, which are absent in all these papers. Furthermore, our paper contrasts with Austen-Smith and Wright (1992) and Rasmusen (1993) in that the latter two take as exogenously given the cost for the PM to verify an IG's information, while we endogenize it by taking account of the access constraint. More specifically, the cost of awarding access to an IG and verifying its information is, in our setting, an opportunity cost that corresponds to the value of the information the PM could obtain by instead granting access and verifying the information of another IG.

Milgrom (1981) and Milgrom and Roberts (1986) introduce games of persuasion, in which a special interest endowed with information seeks to influence a decision maker by choosing which pieces of information to convey truthfully to the decision maker and which pieces of information to withhold from him.[8] We adopt the framework of persuasion games, modelling IGs' decisions whether to withhold information (by not lobbying) or to reveal it truthfully (when lobbying and being granted access). Lagerlöf (1997) considers a game of persuasion in which an IG chooses to collect verifiable information and whether to reveal or to withhold it from a PM. Like us, Lagerlöf identifies a possibility for a Pareto improvement. However, the nature and the source of the Pareto improvement are different. In Lagerlöf (1997) the source of the Pareto improvement lies in: 1) the PM's inability to observe directly the information collection decision of the IG, resulting in the IG collecting on average too much information; and 2) the PM caring about the IG's payoff. In this context, banning informational lobbying has therefore the potential to generate a Pareto improvement. By contrast, our PM knows that IGs are informed (although he does not know which information IGs have). In our paper, the source of the Pareto improvement lies instead in the precision of the information contained in the lobbying decision of each IG. Introducing a constraint on the agenda can improve information transmission and, in circumstances we identify

[7]Some scholars argue that deception rarely occurs in practice. This is because, as Hansen (1991), Berry (1997) and Ainsworth (2002) observe, IGs must build and preserve a reputation for reliability in order to keep access to lawmakers and maintain their ability to influence policy choices.

[8]Brocas and Carrillo (2007), Brocas, Carrillo and Palfrey (2012), Kamenica and Gentzkow (2011) and Gul and Pesendorfer (2012) are relatively recent papers presenting models of persuasion in which a special interest decides how much public information to produce before a decision maker takes an action. Beyond considering a policy choice on a single issue, these papers differ from ours in that they consider settings where information is public and, therefore, symmetric. In our setting, information is private to the IGs and, therefore, asymmetric.

below, generates a Pareto improvement.[9]

Our paper is also related to a small literature on lobbying and access. Austen-Smith (1995, 1998) and Cotton (2009, 2012) consider models in which IGs make monetary contributions, not in exchange for policy favors, as considered in many contributions on IG influence,[10] but instead in the hope of securing access and present their information to the PM.[11,12] All these papers differ from ours in two related ways. First, all these papers endogenize the cost of access which takes the form either of monetary contributions that IGs choose to make to the PM in the hope of securing access, or of an access price set by the PM. By contrast, in our setting the cost that an IG must incur in the hope of gaining access to the PM takes the form of a lobbying cost that is exogenously given (e.g., the cost of setting an office in D.C. or hiring a lobbyist); hence, in our setting, there is no payment of monetary contributions to the PM. Second, in all these papers access has a monetary value for the PM while, in our setting, access has only an informational value since IGs make no monetary contributions to the PM.

Finally, our paper is related to a vast strand of literature in which a PM (or, more generally, a decision maker) chooses how to allocate scarce time and resources (e.g., Holmstrom and Milgrom (1991)). Esteban and Ray (2006) considers a model in which a PM must allocate a limited number of licenses to firms that differ in their productivity and their wealth. Wealth and productivity are private information. Firms seek to signal their productivity by devoting resources toward lobbying. However, the double-dimensional information asymmetry, on productivity and on wealth, can jam the signal since wealthier firms are able to devote more resources toward lobbying than less-wealthy firms of equal, or higher, productivity. As in our paper, Esteban and Ray consider a multidimensional policy choice, with a constraint on the agenda, and find that IG influence can lead to inefficient policymaking. However, the source of inefficiency is different in the two papers. In Esteban and Ray (2006), the inefficiency comes from the double-dimensional information asymmetry, on productivity and on wealth, which is not present in our paper. By contrast, in our paper, the inefficiency comes from the access constraint that induces IGs to overlobby, thereby jamming the signal contained in lobbying decisions, which is not present in Esteban and Ray (2006) where the PM cannot grant access to IGs.

Cotton and Dellis (2016) considers a model of informational lobbying in which

[9] Bennedsen and Feldmann (2002, 2006), Yu (2005) and Dahm and Porteiro (2008a, 2008b) are other papers on IG influence that adopt the framework of persuasion games. These papers differ from ours in several important ways. First, they all consider a single IG which can always get access and convey its information if it chooses to do so, thereby ignoring the constraint on access which is at the heart of our analysis. Second, Bennedsen and Feldmann (2002) considers policymaking in a legislative assembly, not by a single PM as we do. Third, Yu (2005), Bennedsen and Feldmann (2006) and Dahm and Porteiro (2008a, 2008b) all study the IG's choice between conveying information and/or making monetary contributions in exchange for policy favors. The possibility for monetary contributions is absent from our setting.

[10] See, for example, Grossman and Helpman (2002) for a review of the vast literature that models IG influence as quid pro quo exchanges of monetary contributions for policy favors.

[11] These papers differ, among other things, in the nature of information they consider. In Austen-Smith (1995) information is non-verifiable. In Cotton (2009) information can be withheld from the PM but cannot be tampered with. In Austen-Smith (1998) and Cotton (2012), as in our paper, information consists of hard evidence that can be neither tampered with nor withheld once access is granted.

[12] Langbein (1986) and Wright (1990), among others, provide evidence that is consistent with the idea that monetary contributions by IGs serve to buy access, with the purpose of presenting information to lawmakers.

a PM can reform only a limited number of issues. However, the PM is uninformed about which issues are the most pressing and/or beneficial to reform, but can exert effort to learn about the alternative reforms before choosing policy. Next to the PM there are single-issue-minded IGs, which can likewise collect information about the merits of reforming their respective issue.[13] Cotton and Dellis show that even pure informational lobbying (viz. in the absence of monetary contributions and evidence distortion or withholding) can be harmful to citizens' welfare since IGs advocating a priori less-appealing reforms have an incentive to collect information on their reform. IGs' information collection has the effect of weakening the PM's incentives to learn about other issues and can induce him to shift his priorities to issues that are less important for his constituents. As in our paper, Cotton and Dellis consider a multidimensional policy choice with a constraint on the agenda, and find that informational lobbying can lead to inefficient policymaking. However, the source of inefficiency is different in the two papers. In Cotton and Dellis (2016), where information is symmetric, inefficiency comes from the effect that IG information collection has on the PM's own information collection decision. In our paper, where the PM cannot collect information on his own, inefficiency comes from the access constraint and the information asymmetry between the PM and the IGs.

Less closely related to our paper are contributions that look at the effect of elections on politicians' decisions on how to allocate scarce resources. Dellis (2009) shows how electoral concerns can induce an incumbent to address a different set of issues than he would in the absence of electoral concerns; his purpose is to manipulate the set of issues that will be salient in the next election. Daley and Snowberg (2014) shows how candidates, whose abilities are private information, may devote a limited amount of time toward fund-raising rather than policymaking, in an effort at signalling their ability.

3. BASELINE MODEL

We develop our argument using a simple model of access. In section 6 and the supplementary online appendix we generalize our argument and investigate the robustness of our conclusions.

We consider a PM who must choose policy on two issues, indexed by $i = 1, 2$.[14] We denote a policy by $p = (p_1, p_2)$, where $p_i \in \{0, 1\}$ is the policy on issue i. Policy $p_i = 1$ corresponds to the adoption of a reform project or the realization of a public investment on issue i. Policy $p_i = 0$ corresponds to keeping the status quo.

There are two possible states of the world on each issue. We denote the state on issue i by $\theta_i \in \{0, 1\}$. State $\theta_i = 1$ corresponds to circumstances in which the PM benefits from reforming issue i. State $\theta_i = 0$ corresponds to circumstances in which the PM benefits from keeping the status quo on this issue. States are independent across issues. The realized state $\theta = (\theta_1, \theta_2)$ is unknown to the PM, but its distribution is common knowledge:

$$\theta_i = \begin{cases} 1 & \text{with probability } \pi_i \in \left(0, \frac{1}{2}\right) \\ 0 & \text{with probability } 1 - \pi_i. \end{cases}$$

[13] Ellis and Groll (2014) extends this framework by allowing IGs to make legislative subsidies that relax the agenda constraint of the PM.

[14] An issue can be interpreted literally (e.g., gun control, same-sex marriage) or as a public investment project (e.g., the construction of a new bridge or a new sports arena).

Given policy $p = (p_1, p_2)$ and state $\theta = (\theta_1, \theta_2)$, the PM gets utility

$$U(p, \theta) = \alpha \cdot u_1(p_1, \theta_1) + u_2(p_2, \theta_2)$$

where $\alpha > 1$ represents the importance of issue 1 relative to issue 2,[15] and

$$u_i(p_i, \theta_i) = \begin{cases} 1 & \text{if } p_i = \theta_i \\ 0 & \text{otherwise} \end{cases}$$

represents the PM's utility over issue i. Hence, for each issue i, the PM prefers that the policy p_i coincides with the realized state θ_i, which makes information on θ_i valuable to the PM.

There are two IGs, each one representing a separate issue. Given policy $p = (p_1, p_2)$, the IG involved with issue i (henceforth, IG_i) gets utility $v_i(p_i) = p_i$, meaning that IG_i seeks to maximize the probability that $p_i = 1$, regardless of the state θ_i.

Each IG_i has verifiable evidence about θ_i, and decides whether to lobby the PM.[16] If IG_i decides to lobby, it bears a utility cost $f_i \in (0, 1)$ and, if granted access by the PM, must reveal θ_i.[17,18] The PM faces a time constraint that prevents him from granting access to more than one IG.

We are interested in studying the implications of a second constraint, namely, a constraint on the agenda.[19] For this purpose, we shall compare two games, one in which the PM is not constrained on his agenda and can implement both reform projects if he wishes to, and another game in which the PM is constrained on his agenda and can implement only one reform project. We denote by $N \in \{1, 2\}$ the maximum number of reform projects the PM can adopt. We shall call N-game the game in which the PM can adopt up to N reform projects.

The policymaking process has four stages. At stage 0, Nature chooses the state θ_i for each issue i and reveals it only to IG_i. At stage 1 IGs decide simultaneously whether or not to lobby the PM. At stage 2 the PM observes IGs' lobbying decisions and chooses to which IG, if any, he grants access. If IG_i is granted access, it must reveal θ_i to the PM. Finally, at stage 3 the PM chooses policy. We now describe the structure of each stage, working backwards.

[15] Assuming $\alpha > 1$ is made to simplify exposition. Similar qualitative conclusions are obtained if we allow for $\alpha = 1$.

[16] Lobbying can be interpreted in several ways. First, it can be interpreted as the IG knocking at the door of the PM or calling him in order to secure an appointment or a spot at a committee hearing at which the IG will be able to present evidence on its issue. Alternatively, lobbying can be interpreted as the IG preparing a report on its issue and bringing the report to the PM for him to read. In the latter interpretation, access can be interpreted as the decision by the PM to read the report provided by the IG.

[17] Conclusions are qualitatively similar if lobbying is costless but there is a penalty f_i that IG_i must bear if it lobbies and is granted access when $\theta_i = 0$ (e.g., f_i would be a reputation cost).

[18] Thus, when granted access, an IG cannot hide or distort evidence. For example, in some countries witnesses must take an oath before testifying in front of a legislative committee. Results are similar if an IG can withhold, but not alter, evidence; in equilibrium an IG with favorable evidence would choose to reveal it with probability one, when granted access (Milgrom and Roberts (1986)).

[19] For example, a budget constraint may prevent the PM from realizing the public investment project on each issue, and must therefore choose which one of the two investment projects (if any) he will realize. Alternatively, a time constraint may prevent the PM from addressing every single issue landing on his desk, forcing him to ignore at least one of the two issues. Jones and Baumgartner (2005) and Baumgartner et al. (2009), among others, provide detailed accounts of the agenda constraint that lawmakers are facing.

3.1. Stage 3: Policy choice

By the time the PM chooses policy, he has observed the lobbying decisions of the IGs and the realized state for the issue advocated by the IG that was granted access (if any). We denote IG$_i$'s lobbying decision by ℓ_i, where $\ell_i = 1$ if IG$_i$ has lobbied and $\ell_i = 0$ otherwise.[20] We denote the PM's access decision with respect to IG$_i$ by a_i, where $a_i = 1$ if the PM has granted access to IG$_i$ and $a_i = 0$ otherwise. Given profiles of lobbying decisions $\ell = (\ell_1, \ell_2)$ and of access decisions $a = (a_1, a_2)$, the PM forms belief $\beta_i(\ell_i, a_i; \theta_i)$ that $\theta_i = 1$, using Bayes' rule whenever possible. To lighten notation, we write β_i as a shorthand for $\beta_i(\ell_i, a_i; \theta_i)$ when this does not create confusion.

A policy choice strategy for the PM is a mapping

$$\rho : \{0,1\}^2 \times \{0,1\}^2 \to [0,1]^2$$

where $(\rho_1, \rho_2)(\ell, a)$ specifies the probability that the PM chooses policy $p_1 = 1$ and policy $p_2 = 1$, respectively.[21]

The PM maximizes his expected utility with policy choice strategy

$$\rho_i(\ell, a) \begin{cases} = 1 & \text{if } \beta_i > 1/2 \\ \in [0,1] & \text{if } \beta_i = 1/2 \\ = 0 & \text{if } \beta_i < 1/2 \end{cases}$$

when $N = 2$, and

$$\rho_i(\ell, a) \begin{cases} = 1 & \text{if } \beta_i > 1/2 \text{ and } \left(\beta_i - \frac{1}{2}\right) \cdot \alpha_i > \left(\beta_{-i} - \frac{1}{2}\right) \cdot \alpha_{-i} \\ \in [0,1] & \text{if } \beta_i \geq 1/2 \text{ and } \left(\beta_i - \frac{1}{2}\right) \cdot \alpha_i \geq \left(\beta_{-i} - \frac{1}{2}\right) \cdot \alpha_{-i} \\ = 0 & \text{otherwise} \end{cases}$$

when $N = 1$, where $\alpha_1 \equiv \alpha$ and $\alpha_2 = 1$, and with the restriction that $\sum_{i=1}^{2} \rho_i(\ell, a) \leq 1$ when $N = 1$.

Thus, when there is no agenda constraint ($N = 2$) the PM adopts the reform on issue i if he believes $\theta_i = 1$ is more likely than $\theta_i = 0$. When there is an agenda constraint ($N = 1$) the PM adopts the reform on issue i if, again, he believes that $\theta_i = 1$ is more likely than $\theta_i = 0$ and, moreover, the expected utility gain from adopting the reform on issue i exceeds the expected utility gain from adopting the reform on the other issue. The latter implies that when $N = 2$ the PM's policy choice on issue i depends on his belief (and thus on his information) on only θ_i, while when $N = 1$ it depends on his beliefs (and thus on his information) on both θ_1 and θ_2.

3.2. Stage 2: Access

The PM chooses whether to grant access to an IG and, if so, which one. In the baseline model we consider the case where the PM can grant access to an IG only if it did lobby.

[20] Alternatively, we could model each IG$_i$'s lobbying decision as sending a message $m_i \in \{0, 1, \emptyset\}$, where $m_i = 0$ (resp. $m_i = 1$) would correspond to a message that $\theta_i = 0$ (resp. $\theta_i = 1$) and $m_i = \emptyset$ would correspond to the situation where IG$_i$ does not bear the lobbying cost f_i and sends no message. This would slightly complicate the analysis while generating the same conclusions since, in equilibrium, no IG chooses to bear the lobbying cost to send a message $m_i = 0$.

[21] In order to lighten notation, we omit strategies' dependency on N, the maximum number of reform projects the PM can adopt. Likewise, we omit θ from the notation describing PM's posterior beliefs, β_is, and policy choices, ρ_is.

By the time the PM chooses to which IG he grants access, he has observed the lobbying decisions of the two IGs. Given IG_i's lobbying decision ℓ_i, the PM forms belief $\beta_i^{Acc}(\ell_i)$ that $\theta_i = 1$, where the superscript Acc stands for access stage. To lighten notation we write β_i^{Acc} as a shorthand for $\beta_i^{Acc}(\ell_i)$ when this does not create confusion.

An access strategy for the PM is a mapping

$$\gamma : \{0,1\}^2 \to [0,1]^2$$

where $(\gamma_1, \gamma_2)(\ell)$ specifies the probability that the PM grants access to IG_1 and to IG_2, respectively, given a profile of lobbying decisions $\ell = (\ell_1, \ell_2)$.

The PM's access decision is straightforward if at least one IG did not lobby. If neither IG lobbied, the PM cannot grant access to any of them, and so $\gamma_1(0,0) = \gamma_2(0,0) = 0$. If only one IG did lobby, the PM can grant access only to that IG, in which case we have $\gamma_1(1,0) = \gamma_2(0,1) = 1$ and $\gamma_1(0,1) = \gamma_2(1,0) = 0$.

We now consider the more interesting case where the two IGs did lobby (i.e., $\ell = (1,1)$). In this case, the PM chooses an access strategy that solves

$$\begin{cases} \max\limits_{(\gamma_1,\gamma_2)\in[0,1]^2} & EU(\gamma \mid \rho) \\ \text{s.t.} & \gamma_1 + \gamma_2 = 1 \end{cases}$$

where

$$EU(\gamma \mid \rho) \equiv \sum_{i=1}^{2} \left[\gamma_i \cdot W_i(\rho) + (1-\gamma_i) \cdot Z_i(\rho)\right] \cdot \alpha_i$$

is the PM's expected utility. $W_i(\rho)$ denotes the probability with which the PM believes he will make the correct policy choice on issue i *if he grants access* to IG_i, and $Z_i(\rho)$ denotes the same probability *if he does not grant access* to IG_i. The expressions for $W_i(\rho)$ and $Z_i(\rho)$ are given in a supplementary online appendix. We define $X_i(\rho) \equiv W_i(\rho) - Z_i(\rho)$ as the increase in the probability that the PM will make the correct policy choice on issue i by granting access to IG_i.

The PM's access strategy when both IGs lobby is given by

$$\gamma_i(1,1) \begin{cases} = 1 & \text{if } X_i(\rho) \cdot \alpha_i > X_{-i}(\rho) \cdot \alpha_{-i} \\ \in [0,1] & \text{if } X_i(\rho) \cdot \alpha_i = X_{-i}(\rho) \cdot \alpha_{-i} \\ = 0 & \text{if } X_i(\rho) \cdot \alpha_i < X_{-i}(\rho) \cdot \alpha_{-i} \end{cases}$$

with the restriction that $\sum_{i=1}^{2} \gamma_i(1,1) = 1$. Thus, when both IGs lobby, the PM chooses to grant access to the IG which information has the highest expected value for him.

3.3. Stage 1: Lobbying

A lobbying strategy for IG_i is a mapping

$$\lambda_i : \{0,1\} \to [0,1]$$

where $\lambda_i(\theta_i)$ specifies the probability that IG_i lobbies the PM.

IG_i chooses a lobbying strategy that solves

$$\max_{\lambda_i(\theta_i)\in[0,1]} Ep_i(\lambda_i(\theta_i), \lambda_{-i}, \gamma, \rho) - \lambda_i(\theta_i) \cdot f_i$$

where $Ep_i(\lambda_i(\theta_i), \lambda_{-i}, \gamma, \rho)$ is the probability that $p_i = 1$.

3.4. Equilibrium

The solution concept is Perfect Bayesian equilibrium. Roughly speaking, an equilibrium consists of a strategy profile $\{\lambda(\cdot), \gamma(\cdot), \rho(\cdot)\}$ and a system of beliefs $\left\{\beta^{Acc}(\cdot), \beta(\cdot)\right\}$ such that: 1) the strategy profile is sequentially rational given the system of beliefs; and 2) the beliefs are obtained from the strategies using Bayes' rule whenever possible.

The finiteness of the game implies that an equilibrium exists in each of the two games, the $N = 1$-game and the $N = 2$-game. In case of equilibrium multiplicity, we shall focus attention on most informative equilibria, a standard refinement in the literature.

3.5. Extensions

In order to make our argument as simple as possible, we have made a series of assumptions. In section 6 and a supplementary online appendix we generalize our argument along several dimensions and investigate the robustness of our conclusions. First, we consider a setting with an arbitrary, finite number of issues $I \geq 2$. Second, we endogenize the set of IGs by adding a preliminary stage at which each IG decides whether to organize or, equivalently, whether to acquire information about the realized state for its issue. Third, we grant the PM with subpoena power, allowing him to award access to an IG whether it did lobby or not.

4. AN ILLUSTRATIVE EXAMPLE

In this section we present the main qualitative results of our model by means of an illustrative example. The example characterizes equilibrium for the game without agenda constraint ($N = 2$) and for the game with agenda constraint ($N = 1$). We show, for some parameters values, that agenda constraint results in better information transmission in equilibrium. Specifically, we show that agenda constraint leads to truthful lobbying by the IGs, leading to the PM becoming fully informed, whereas placing no constraint on the agenda leads to an equilibrium in which there is "overlobbying" which prevents the PM from becoming fully informed.[22] Moreover, we show that the equilibrium outcome of the game with agenda constraint Pareto-dominates the equilibrium outcome of the game without agenda constraint.

Here we provide the numerical results to highlight these points along with the intuition behind our derivations; detailed calculations can be found in the supplementary online appendix. Consider the following parameters values:

- $\alpha = 2$, i.e., the PM finds issue 1 twice as important as issue 2,
- $\pi_1 = \pi_2 = 2/5$, i.e., the PM is ex ante biased against reforms (since $\pi_i < 1/2$), and
- the lobbying cost for each IG is $f = 1/20$.

4.1. Game without agenda constraint ($N = 2$)

We will first establish that any equilibrium of the game without agenda constraint cannot lead to full information revelation. To see this, assume by way of

[22] It is clear that both IGs lobbying truthfully is sufficient to obtain full information revelation. However, a reader may wonder whether truthful lobbying is necesary for full information revelation. We will formally establish in section 5 that it is the case in equilibrium.

contradiction that such an equilibrium were to exist. In this case lobbying must be truthful, as we will show in section 5. Let $\widehat{\gamma}_i \in [0,1]$ denote the equilibrium probability with which IG_i is granted access when both IGs lobby, with the restriction that $\widehat{\gamma}_1 + \widehat{\gamma}_2 = 1$. Given truthful lobbying, in equilibrium the PM's interim beliefs are such that if IG_i lobbies (does not lobby), then he believes $\theta_i = 1$ (0). It follows that, if an IG lobbies but is not granted access, the PM chooses to reform its issue. IG_i's expected policy payoff from lobbying when $\theta_i = 0$ is then equal to $2/5 \cdot \widehat{\gamma}_{-i}$, which corresponds to the probability that IG_{-i} lobbies and is granted access. For IG_i to not deviate and lobby when $\theta_i = 0$, it must be that $2/5 \cdot \widehat{\gamma}_{-i} \leq 1/20$, i.e., the expected policy gain from lobbying must not exceed the lobbying cost. This inequality is satisfied for each $i = 1, 2$ only if $\widehat{\gamma}_i \leq 1/8$ for $i = 1, 2$, which contradicts $\widehat{\gamma}_1 + \widehat{\gamma}_2 = 1$.

Next, we assert that there is a unique equilibrium of the game without agenda constraint which has the following strategies and beliefs:[23]

1. The lobbying strategies are given by $\lambda_i(1) = 1$ for each $i = 1, 2$, $\lambda_1(0) = 2/9$ and $\lambda_2(0) = 2/3$, i.e., IG_i always lobbies when it has favorable information and randomizes between lobbying and not lobbying when it has unfavorable information.
2. The access strategy is such that when both IGs lobby, $\gamma_1(1,1) = 15/16$ and $\gamma_2(1,1) = 1/16$, i.e., the PM randomizes between granting access to the IGs; IG_1 is granted access with a (much) higher probability. When only one IG lobbies, it is granted access with probability one. When neither IG lobbies, the PM cannot grant access to any IG.
3. The policy strategy is such that the PM chooses $p_1 = 1$ when IG_1 lobbies and is not granted access. The PM chooses $p_2 = 1$ with probability 1/10 when IG_2 lobbies and is not granted access. Finally, $p_i = 0$ when IG_i does not lobby, and $p_i = \theta_i$ when IG_i lobbies and is granted access.
4. PM's beliefs at the access stage are obtained from the lobbying strategies using Bayes' rule: $\beta_i^{Acc}(0) = 0$ for each $i = 1, 2$, $\beta_1^{Acc}(1) = \frac{2/5}{2/5+(3/5)\cdot(2/9)} = 3/4$ and $\beta_2^{Acc}(1) = \frac{2/5}{2/5+(3/5)\cdot(2/3)} = 1/2$.

To get the intuition behind why the above strategies and beliefs constitute an equilibrium, let's focus on the PM's optimal actions when both IGs lobby.[24] IG_1's lobbying strategy is such that whenever IG_1 lobbies, the PM believes with a sufficiently high probability that $\theta_1 = 1$, and therefore, if IG_1 were not granted access, the PM implements reform on issue 1. On the other hand, lobbying strategy of IG_2 is not sufficiently informative, and therefore, if IG_2 were not granted access, the PM is indifferent between implementing and not implementing reform on issue 2. Hence, when both IGs lobby, the PM has two strategies to consider: 1) grant access to IG_1, choose the correct policy on issue 1 and randomize between reform and status quo on issue 2; 2) grant access to IG_2, choose the correct policy on issue 2 and implement reform on issue 1. Given the parameters values and the lobbying frequencies chosen in equilibrium, it can be shown that these two strategies yield identical payoff, 5/2. It is therefore optimal for the PM to randomize between the two, specifically, to choose $\gamma_1(1,1) = 15/16$ and $\gamma_2(1,1) = 1/16$.

We can then verify that, given the mixed access and policy strategies employed

[23] See lemma 1 in the next section for formal statement and proof.

[24] This is the interesting case. The other cases are as follows: when an IG_i does not lobby, the PM rationally infers that $\theta_i = 0$ and therefore chooses to maintain the status quo. When exactly one IG lobbies, the PM grants it access, learns the true state, and acts optimally according to his information.

by the PM, each IG_i prefers to lobby when $\theta_i = 1$ and is indifferent between lobbying and not lobbying when $\theta_i = 0$, which makes it optimal to play a mixed lobbying strategy in state 0, specifically, $\lambda_1(0) = 2/9$ and $\lambda_2(0) = 2/3$.

We calculate the equilibrium ex-ante expected payoffs of the two IGs and the PM and can show them to be the following:

$$Ev_1^{N=2} = \frac{19}{50},\ Ev_2^{N=2} = \frac{1}{5},\ EU^{N=2} = \frac{209}{75}.$$

4.2. Game with agenda constraint ($N = 1$)

We now show that there exists an equilibrium for the game with agenda constraint ($N = 1$) in which the PM gets perfectly informed about θ. (This is true in any equilibrium, as established in lemma 2.) This illustrates our result that an agenda constraint can lead to better information transmission (proposition 1).

Suppose the IGs choose truthful lobbying strategies, i.e., for $i = 1, 2$, $\lambda_i(1) = 1$ and $\lambda_i(0) = 0$. Suppose the PM chooses $\gamma_1(1, 1) = 1$, i.e., he grants access to IG_1 when both IGs lobby.[25] If both IGs lobby and $\theta_1 = 1$, then the PM chooses reform only on issue 1; if both IGs lobby and $\theta_1 = 0$, then the PM implements reform only on issue 2.

To show that these strategies constitute an equilibrium, it remains to establish that truthful lobbying is an optimal strategy for both IGs. Observe that issue i does not get reformed when $\theta_i = 0$, implying that IG_i has no incentive to deviate and lobby in this state. When $\theta_1 = 1$, IG_1 gets payoff $1 - 1/20 = 19/20$ from lobbying (i.e., it gets $p_1 = 1$ and must bear lobbying cost $f = 1/20$). When $\theta_2 = 1$, IG_2 gets expected payoff $3/5 - 1/20 = 11/20$ from lobbying (i.e., it gets $p_2 = 1$ when $\theta_1 = 0$, which happens with probability 3/5, and must bear lobbying cost $f = 1/20$). Since each IG gets zero payoff if it does not lobby ($p_i = 0$ since $\beta_i^{Acc}(0) = 0$), neither IG_i wants to deviate and not lobby when $\theta_i = 1$.

To sum up, through lobbying decisions the PM gets perfectly informed about θ. He always chooses the correct p_1. He also chooses the correct p_2 unless $\theta = (1, 1)$, in which case the agenda constraint is binding and the PM reforms issue 1, while keeping the status quo for issue 2. We show equilibrium expected payoffs for the PM and the IGs to be

$$Ev_1^{N=1} = \frac{19}{50},\ Ev_2^{N=1} = \frac{11}{50},\ EU^{N=1} = \frac{71}{25}.$$

4.3. Pareto improvement

Comparing equilibrium expected payoffs in the two games, we get

$$\begin{aligned} Ev_1^{N=2} &= \frac{19}{50} = Ev_1^{N=1} \\ Ev_2^{N=2} &= \frac{1}{5} < \frac{11}{50} = Ev_2^{N=1} \\ EU^{N=2} &= \frac{209}{75} < \frac{71}{25} = EU^{N=1}. \end{aligned}$$

Thus, IG_1 is ex ante as well off in the $N = 1$-game as in the $N = 2$-game, while IG_2 and the PM are each ex ante strictly better off in the former than in the latter. This

[25] The access and policy choice strategies when none or exactly one IG lobbies are same as in the previous case.

illustrates our second result that, from an ex ante point of view, the introduction of an agenda constraint can generate a Pareto improvement (proposition 2).

Note that it is not *a priori* clear that introduction of an agenda constraint could lead to Pareto improvement, nor does it lead to such improvement for all parameter values. Intuitively, there are costs and benefits of imposing an agenda constraint for the PM as well as the IGs. The downside is that agenda constraint reduces the choice set of the PM, and therefore reduces the chance that a reform is implemented. On the positive side, agenda constraint can provide the PM with a tool to credibly discipline the IGs to truthfully reveal information through their lobbying decisions. This may not only benefit the PM but may also benefit the IGs by saving them the costs associated with overlobbying. Under a range of parameter values, the benefits are greater than the costs for *all* three players, giving us Pareto improvement. In section 5.3 we provide the precise conditions under which this occurs.

5. GENERAL RESULTS

We now analyze the baseline model described in section 3. First, we describe the equilibrium sets in the two games, the $N = 1$-game and the $N = 2$-game. Second, we compare equilibrium information transmission in the two games. Finally, we identify the region of the parameter space where the introduction of an agenda constraint generates a Pareto improvement.

5.1. Equilibrium sets

In this subsection, we characterize the (most informative) equilibrium in each game.

5.1.1. Game without agenda constraint

We start by considering the game without agenda constraint ($N = 2$). As we showed in section 3, the policy choice on issue i depends only on the PM's belief on that issue, $\beta_i(\ell_i, a_i)$. Given this observation, and to lighten notation, we shall write $\rho_i(\ell_i, a_i)$ in place of $\rho_i(\ell, a)$.

Our first lemma describes equilibria of this game.[26]

LEMMA 1. *Consider the $N = 2-$game.*

1. *If $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} \geq 1$, an equilibrium exists in which $\lambda_i(1) = 1$ and $\lambda_i(0) = 0$ for each $i = 1, 2$.*
2. *If $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$, there is a unique equilibrium. In this equilibrium, we have*

$$\begin{cases} \lambda_i(1) = 1 \text{ and } \lambda_i(0) = \frac{\pi_i}{1-\pi_i} \frac{1}{2\alpha_i - 1}, \text{ where } \alpha_1 \equiv \alpha \text{ and } \alpha_2 = 1 \\ \gamma_2(1,1) = 1 - \gamma_1(1,1) = \frac{f_1}{2\pi_2} \\ \rho_i(0,0) = 0,\ \rho_1(1,0) = 1 \text{ and } \rho_2(1,0) = \frac{\pi_2 \cdot (2\alpha - 1) \cdot f_2}{\pi_1 \alpha \cdot (2\pi_2 - f_1)} \\ \beta_i(0,0) = \beta_i^{Acc}(0) = 0 \text{ and } \beta_1(1,0) = \beta_1^{Acc}(1) > \frac{1}{2} = \beta_2(1,0) = \beta_2^{Acc}(1) \end{cases}$$

for $i = 1, 2$.

[26] In an effort to save space, we report here only the strategies and beliefs that are needed to determine the policy outcome and the payoffs of the players. Detailed equilibrium strategies and beliefs are specified in the proof of the lemma, which can be found in the appendix.

Thus, truthful lobbying can be supported in equilibrium of the $N = 2$-game if and only if lobbying is sufficiently costly, in the sense that $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} \geq 1$. To see why, suppose both IGs lobby truthfully. At the access stage, the PM believes $\theta_i = 1$ (resp. $\theta_i = 0$) if he observes IG$_i$ lobbying (resp. not lobbying). For truthful lobbying to be supported in equilibrium, IG$_i$ must not want to deviate and lobby when $\theta_i = 0$. If IG$_i$ does not lobby, it cannot be granted access, and the PM believes $\theta_i = 0$ and chooses $p_i = 0$. If IG$_i$ lobbies when $\theta_i = 0$, the PM chooses $p_i = 1$ if and only if he grants access to the other IG, IG$_{-i}$. This event occurs with probability $\pi_{-i} \cdot \gamma_{-i}(1,1)$, viz. the probability that IG$_{-i}$ lobbies times the probability that IG$_{-i}$ is granted access when both IGs lobby. Thus, IG$_i$ does not deviate from its truthful lobbying strategy when $\theta_i = 0$ if and only if the expected gain does not exceed the lobbying cost, i.e., $\pi_{-i} \cdot \gamma_{-i}(1,1) \leq f_i$. Recalling that $\gamma_1(1,1) + \gamma_2(1,1) = 1$, this condition is satisfied for each IG$_i$ if and only if

$$\gamma_1(1,1) \in \left[1 - \frac{f_1}{\pi_2}, \frac{f_2}{\pi_1}\right].$$

This interval is non-empty if and only if $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} \geq 1$.

When lobbying is sufficiently costly to support truthful lobbying in equilibrium (case 1 of lemma 1), lobbying decisions reveal θ and the PM chooses $p = \theta$.

When lobbying is not sufficiently costly to support truthful lobbying in equilibrium (case 2 of lemma 1), there is a unique equilibrium. In this equilibrium, each IG overlobbies, i.e., lobbies with probability one when it has favorable information, and randomizes between lobbying and not lobbying when it has unfavorable information. An IG randomizes between lobbying and not lobbying only if it is indifferent between taking any of these two actions. Observe that an IG$_i$ gets zero payoff if it does not lobby; this happens because $\beta_i^{Acc}(0) = 0$, since IG$_i$ abstains from lobbying only when $\theta_i = 0$, which induces the PM to choose $p_i = 0$. This means that IG$_i$ must get zero expected payoff when it lobbies in state $\theta_i = 0$. This occurs if IG$_i$ lobbies with a probability such that, in expectation, all the rent from overlobbying is exhausted.

In the case of IG$_2$, this imposes two requirements. A first requirement is that the PM is indifferent between choosing $p_2 = 1$ and $p_2 = 0$ when IG$_2$ lobbies but is not granted access. This happens if and only if $\beta_2^{Acc}(1) = 1/2$, which pins down a precise value for $\lambda_2(0)$. A second requirement is that the PM randomizes in a precise way between $p_2 = 1$ and $p_2 = 0$, which pins down a precise value for $\rho_2(1,0)$.

In the case of IG$_1$, this imposes that, when both IGs lobby, the PM randomizes in a precise way between granting access to IG$_1$ and granting access to IG$_2$. This pins down a precise value for $\gamma_1(1,1)$. Moreover, the PM randomizes on access if and only if the value of the information he expects to get from IG$_1$ is the same as the value of the information he expects to get from IG$_2$. Given $\beta_2^{Acc}(1) = 1/2$, the expected value of the information from IG$_2$ equals $1/2$. At the same time, the expected value of the information from IG$_1$ equals $\left[1 - \beta_1^{Acc}(1)\right] \cdot \alpha$.[27] The condition $\left[1 - \beta_1^{Acc}(1)\right] \cdot \alpha = 1/2$ pins down an exact value for $\beta_1^{Acc}(1)$ and, in turn, for $\lambda_1(0)$. Furthermore, we can infer from this equality that $\beta_1^{Acc}(1) > 1/2$, given $\alpha > 1$ and $\beta_1^{Acc}(1) \geq 1/2$, the latter since otherwise IG$_1$ would not want to lobby when $\theta_1 = 0$.[28] It follows from $\beta_1^{Acc}(1) > 1/2$ that $\rho_1(1,0) = 1$.

[27] This corresponds to $X_1(\rho) \cdot \alpha$, as defined in section 3.2.

[28] If IG$_1$ were granted access, it would have to reveal $\theta_1 = 0$. If IG$_1$ were not granted access,

5.1.2. Game with agenda constraint

We continue by considering the game with agenda constraint ($N = 1$). Our second lemma describes equilibrium lobbying strategies in this game.[29]

LEMMA 2. *Consider the $N = 1-$game.*

1. *If $f_2 > 1 - \pi_1$, an equilibrium exists in which lobbying strategies are given by*

$$\begin{cases} \lambda_1(1) = 1 \text{ and } \lambda_1(0) = 0 \\ \lambda_2(1) = \lambda_2(0) = 0. \end{cases}$$

Moreover, in any equilibrium, lobbying strategies are given by

$$\begin{cases} \lambda_i(1) = 1 \text{ and } \lambda_i(0) = 0 \\ \lambda_{-i}(1) = \lambda_{-i}(0) = 0 \end{cases}$$

for some $i \in \{1, 2\}$.

2. *If $f_2 \leq 1-\pi_1$, an equilibrium exists in which $\lambda_i(1) = 1$ and $\lambda_i(0) = 0$ for each $i = 1, 2$. Moreover, if $f_i < 1 - \pi_i$ for each $i = 1, 2$, then in any equilibrium lobbying strategies are given by $\lambda_i(1) = 1$ and $\lambda_i(0) = 0$ for each $i = 1, 2$.*

Thus, equilibria of the $N = 1$-game involve truthful lobbying if lobbying is not too costly (case 2 of lemma 2). To understand the intuition, suppose that both IGs lobby truthfully. In this case, lobbying decisions perfectly reveal θ, which renders the PM indifferent between granting access to IG_1 and granting access to IG_2. Thus, the PM can follow a strategy that prioritizes issue 1, i.e., that grants access to IG_1 whenever it lobbies and that adopts $p_1 = 1$ if and only if IG_1 reveals $\theta_1 = 1$.

Given this strategy of the PM, IG_1 does not want to deviate from truthful lobbying. To see this, note that when IG_1 does not lobby, the PM believes $\theta_1 = 0$ ($\beta_1^{Acc}(0) = 0$) and chooses $p_1 = 0$.

1. When $\theta_1 = 1$, IG_1 does not want to deviate and not lobby since it gets $p_1 = 1$ when lobbying,[30] while it would get $p_1 = 0$ if it were to not lobby.
2. When $\theta_1 = 0$, IG_1 does not want to deviate and lobby since IG_1 would then be granted access and would have to reveal $\theta_1 = 0$, leading the PM to choose $p_1 = 0$, exactly as when IG_1 does not lobby.

Likewise, IG_2 does not want to deviate from truthful lobbying. This happens because IG_2 is awarded access when the PM considers the possibility of adopting $p_2 = 1$. More specifically, when IG_2 does not lobby, the PM believes $\theta_2 = 0$ ($\beta_2^{Acc}(0) = 0$) and chooses $p_2 = 0$.

1. When $\theta_2 = 0$, IG_2 does not want to deviate and lobby since the PM considers adopting $p_2 = 1$ only when IG_1 does not lobby (which, given IG_1's truthful strategy, means that the PM then believes $\theta_1 = 0$). In this case, if IG_2 were to lobby, it would be granted access and would have to reveal $\theta_2 = 0$, leading the PM to choose $p_2 = 0$, exactly as when IG_2 does not lobby. Given that lobbying is costly, IG_2 is then strictly better off not lobbying.

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the PM would believe $\theta_1 = 0$ to be more likely than $\theta_1 = 1$. In either case, the PM would choose $p_1 = 0$, which would be the same policy outcome as if IG_1 were not lobbying. Since lobbying is costly, IG_1 would not want to lobby.

[29] Other equilibrium strategies and beliefs are described in the proof of the lemma.

[30] IG_1 is awarded access with probability one, reveals $\theta_1 = 1$ and, since $\alpha > 1$, the PM chooses $p = (1, 0)$.

2. When $\theta_2 = 1$, if IG_2 lobbies, it gets $p_2 = 1$ with probability $1 - \pi_1$, i.e., the probability that IG_1 does not lobby. Thus, IG_2 does not want to deviate and not lobby if and only if the expected gain from lobbying is at least as large as the lobbying cost, i.e., $f_2 \leq 1 - \pi_1$.

Observe that truthful lobbying implies that, in equilibrium, the PM chooses $p = (1, 0)$ when $\theta = (1, 1)$, and $p = \theta$ otherwise.

Finally, when lobbying is sufficiently costly for IG_2 (case 1 of lemma 2), only one IG lobbies in equilibrium. Being the only lobbying IG, it lobbies truthfully since it expects to be granted access if it lobbies.[31]

5.1.3. Summing up

To sum up, when lobbying costs are sufficiently low, equilibrium lobbying is truthful in the $N = 1$-game, while IGs overlobby in the $N = 2$-game. The key difference between these two games lies in the fact that the agenda constraint allows the PM to better "discipline" the lobbying behavior of IGs by following a strategy where he adopts $p_i = 1$ only if IG_i lobbies and reveals $\theta_i = 1$ when granted access. In the $N = 2$-game, the PM cannot follow such strategies since he would want to deviate and choose $p_{-i} = 1$ when both IGs lobby and he grants access to IG_i, not to IG_{-i}. When lobbying is not too costly, IG_{-i} would then want to deviate and lobby in state $\theta_{-i} = 0$, implying that truthful lobbying cannot be supported.

5.2. Comparing information transmission

In this section we compare equilibrium information transmission in the two games. For this purpose, we compare the PM's equilibrium posterior beliefs (focusing on most-informative equilibria in case of equilibrium multiplicity). We write β_i^N as the PM's equilibrium posterior belief that $\theta_i = 1$ in the N-game. We measure the PM's information about θ_i using $\left|\beta_i^N - 1/2\right|$. This quantity varies between 0 and 1/2. When the PM is perfectly *un*informed about θ_i, we have $\beta_i^N = 1/2$, in which case $\left|\beta_i^N - 1/2\right| = 0$. When the PM is perfectly informed about θ_i, we have $\beta_i^N \in \{0, 1\}$, in which case $\left|\beta_i^N - 1/2\right| = 1/2$.

PROPOSITION 1. *We have:*

1. If $f_2 > 1 - \pi_1$, then

$$\left|\beta_i^{N=2} - 1/2\right| = \frac{1}{2} \geq \left|\beta_i^{N=1} - 1/2\right|$$

with an equality for $i = 1$ and a strict inequality for $i = 2$.

2. If $f_2 \in \left[\pi_1 \cdot \left(1 - \frac{f_1}{\pi_2}\right), 1 - \pi_1\right]$, then

$$\left|\beta_i^{N=2} - 1/2\right| = \frac{1}{2} = \left|\beta_i^{N=1} - 1/2\right|$$

for each $i = 1, 2$.

[31] If $f_2 > 1 - \pi_1$, an equilibrium exists in which IG_1 is the lobbying IG (and, being the only lobbying IG, it does so truthfully). An equilibrium in which IG_2 is the lobbying IG requires restrictive out-of-equilibrium beliefs. As a result, in the rest of the paper we shall focus on the equilibrium in which IG_1 lobbies truthfully and IG_2 does not lobby.

3. If $f_2 < \pi_1 \cdot \left(1 - \frac{f_1}{\pi_2}\right)$, then

$$\left|\beta_i^{N=2} - 1/2\right| \leq \frac{1}{2} = \left|\beta_i^{N=1} - 1/2\right|$$

for each $i = 1, 2$, with a strict inequality for some θ.

Thus, when lobbying is costly for IG_2 (case 1 of proposition 1), the PM gets better informed about θ when $N = 2$ than when $N = 1$. The reverse is true when lobbying is not too costly for IG_2 (case 3 of proposition 1), the PM getting better informed when $N = 1$ than when $N = 2$. For intermediate cases, the PM gets perfectly informed whether $N = 1$ or $N = 2$.

To understand this result, observe that when $N = 2$ the PM gets the better informed the *more* costly lobbying is. This is because a higher lobbying cost helps discipline IGs and prevents excessive lobbying (in the form of overlobbying). By contrast, when $N = 1$ the PM gets better informed the *less* costly lobbying is. This is because a lower lobbying cost helps induce IG_2 to lobby (truthfully) and prevents insufficient lobbying (in the form of IG_2 abstaining from lobbying). The key features underlying this difference between $N = 1$ and $N = 2$ are that an IG's incentives to lobby: 1) decrease with the lobbying cost, and 2) are stronger when $N = 2$ than when $N = 1$.

When $N = 1$, the PM can support truthful lobbying by acting 'lexicographically', awarding access priority to IG_1 and adopting $p_1 = 1$ if and only if IG_1 reveals $\theta_1 = 1$. When lobbying is not too costly, the two IGs lobby truthfully and the PM gets perfectly informed about θ. As lobbying becomes more costly, the net gain of lobbying for IG_2 decreases, up to a point where it becomes negative. Once a threshold is crossed (viz. $1 - \pi_1$), IG_2 stops lobbying, and the PM no longer gets from IG_2 any information on θ_2.

When $N = 2$, the PM cannot follow the same strategies as when $N = 1$. This is because when $N = 2$, the PM can choose to reform both issues, something he cannot do when $N = 1$. When lobbying is costly, even a small probability of being granted access is sufficient to 'discipline' an IG, deterring it from deviating from truthful lobbying. As lobbying becomes less costly, the probability of being granted access must increase to still deter an IG from deviating and lobbying when it has unfavorable information. Once a threshold is crossed (viz. $\pi_1\left(1 - \frac{f_1}{\pi_2}\right)$), it is no longer possible to 'discipline' IGs. IGs overlobby, and the PM is no longer perfectly informed about θ.

To sum up, the PM gets better informed: 1) in the $N = 1$-game, when lobbying costs are low; and 2) in the $N = 2$-game, when lobbying costs are high.

5.3. Pareto-improving agenda constraint

In this section, we identify the region of the parameter space where the introduction of an agenda constraint generates a Pareto improvement.

PROPOSITION 2. *Let EU_k^N denote the equilibrium ex ante expected payoff of player $k \in \{1, 2, PM\}$ in the $N-$game. We have $EU_k^{N=1} \geq EU_k^{N=2}$ for each player $k \in \{1, 2, PM\}$, with at least one inequality strict, if and only if*

$$\frac{\alpha\pi_1 f_1 + (2\alpha - 1) \cdot \pi_2 f_2}{\pi_1\pi_2} \leq 1.$$

To understand this result, observe that an agenda constraint imposes a cost by limiting the choice set of the PM. The introduction of an agenda constraint can therefore generate a Pareto improvement only if it creates a benefit that counterbalances this agenda constraint cost. In our setting, the benefit must be coming from information transmission. We already know from proposition 1 that an agenda constraint leads in equilibrium to a better informed PM if and only if lobbying costs are sufficiently low (case 3 of proposition 1). Thus, only in those circumstances can the PM not suffer from an agenda constraint. We can therefore restrict attention to this case when searching for a Pareto-improving agenda constraint.

We start by observing that IG_1 is ex ante as well off whether there is an agenda constraint or not. This is true for each state θ_1. Specifically,

1. when $\theta_1 = 1$, IG_1 lobbies with probability one and gets $p_1 = 1$ in both games.
2. when $\theta_1 = 0$, IG_1 does not lobby and gets $p_1 = 0$ in the $N = 1$-game. In the $N = 2$-game, IG_1 overlobbies up to the point where it is indifferent between lobbying and not lobbying. In each game IG_1's expected payoff is equal to zero.

We continue by observing that IG_2 is ex ante (weakly) better off with an agenda constraint than without.[32] On the one hand, the agenda constraint creates a cost for IG_2 since the PM prioritizes issue 1, IG_2 thus getting $p_2 = 1$ only if $\theta_1 = 0$. On the other hand, the agenda constraint eliminates a negative externality that IG_2 imposes on itself by overlobbying. To see this, consider each state θ_2, one at a time.

1. When $\theta_2 = 0$, IG_2 gets zero expected payoff whether $N = 1$ or $N = 2$. The logic is similar to the one for IG_1 when $\theta_1 = 0$.
2. Consider the case where $\theta_2 = 1$.
   In the $N = 1$-game, IG_2 gets $p_2 = 1$ only with probability $1 - \pi_1$, i.e., if and only if $\theta_1 = 0$. This reflects the agenda constraint cost imposed to IG_2. Specifically, IG_2 can get its reform adopted only when the PM does not want to adopt the reform on issue 1.
   In the $N = 2$-game, IG_2's overlobbying behavior when $\theta_2 = 0$ induces the PM to adopt $p_2 = 1$ with probability less than one when both IGs lobby and the PM grants access to IG_1, not to IG_2. In other words, IG_2's overlobbying behavior creates a negative externality on IG_2's $\theta_2 = 1$-self by undermining the PM's belief that $\theta_2 = 1$ when IG_2 lobbies but is not granted access.
   The condition stated in proposition 2 is necessary and sufficient for the overlobbying externality cost to exceed the agenda constraint cost.

It remains to consider the PM. On the one hand, the agenda constraint limits the set of feasible policy choices, imposing a cost on the PM. On the other hand, the PM gets better informed about θ when $N = 1$ compared to when $N = 2$, since the agenda constraint allows the PM to 'discipline' the lobbying behavior of IGs and to prevent overlobbying. In this region of the parameter space, the informational benefit exceeds the agenda constraint cost, implying the PM is ex ante strictly better off with agenda constraint than without.

6. EXTENSIONS

This section extends our baseline model in a number of ways. In section 6.1, we extend our analysis to an arbitrary number of issues. In section 6.2, we consider

[32] The indifference occurs if and only if the condition stated in proposition 2 holds with an equality.

a setting in which the PM has subpoena power. In section 6.3, we discuss briefly endogenizing the set of IGs. Proofs for all extension results are provided in a supplementary online appendix.

6.1. More than two issues

In this subsection we will examine whether our main findings from section 5 can be extended to more than two issues. We consider a setting where there is an arbitrary but finite number of issues, I, with one IG on each issue. The PM can reform at most N issues, and can grant access to at most K IGs, where $1 \leq K \leq N \leq I$. To keep our analysis tractable we consider a symmetric setting, i.e., we assume that for each IG$_i$, $\pi_i \equiv \pi < 1/2$, $f_i \equiv f \in (0,1)$ and $\alpha_i \equiv 1$. We will focus on the class of symmetric equilibria, i.e., equilibria in which each IG uses identical lobbying strategies, $\{\lambda^*(\theta_i)\}_{\theta_i \in \{0,1\}}$. The PM's access and policy choices are symmetric with respect to all IGs, i.e., for all i, j such that $\ell_i = \ell_j$, we have $\gamma_i^* = \gamma_j^*$ and for all i, j such that $\ell_i = \ell_j$ and $a_i = a_j$ (and, when $a_i = a_j = 1$, $\theta_i = \theta_j$), we have $\rho_i^* = \rho_j^*$. In a symmetric equilibrium with lobbying strategies $\{\lambda^*(1), \lambda^*(0)\}$ the probability that an IG lobbies is $\delta^* \equiv \pi \cdot \lambda^*(1) + (1-\pi) \cdot \lambda^*(0)$. In addition to this, we impose a reasonable restriction that if PM grants access to IG$_i$ and learns $\theta_i = 1$, then he implements reform on issue i ahead of any other issue j to which access was not granted.[33] The following lemma is useful in characterizing the set of symmetric equilibria.

Lemma 3. *In any symmetric equilibrium the lobbying strategy must be one of the three types:*

1. *Truthful lobbying, i.e.,* $\lambda^*(1) = 1, \lambda^*(0) = 0$ *and hence* $\delta^* = \pi$;
2. *Overlobbying when* $\theta_i = 0$, *i.e.,* $\lambda^*(1) = 1, \lambda^*(0) \in (0,1)$ *and hence* $\delta^* \in (\pi, 1)$; *or*
3. *Underlobbying when* $\theta_i = 1$, *i.e.,* $\lambda^*(1) \in (0,1), \lambda^*(0) = 0$ *and hence* $\delta^* \in (0, \pi)$.

The sketch of proof for the above lemma is as follows. First, observe that IGs never lobbying ($\delta^* = 0$), or always lobbying ($\delta^* = 1$), cannot constitute an equilibrium. In the former case, an IG would prefer to lobby in state 1 since such a deviation will get his reform implemented for sure. In the latter case, the PM's interim belief about a lobbying IG having state 1 must be $\pi < 1/2$, which means a lobbying IG in state 0 will not get reform implemented; hence, it is better off not lobbying since such a deviation will save on the cost of lobbying. Second, observe that in any symmetric equilibrium, the marginal return to lobbying is strictly greater in state $\theta = 1$ than in state $\theta = 0$. This implies that for an IG to be playing a strictly mixed strategy in state 0 (i.e., $\lambda^*(0) \in (0,1)$) it must be indifferent between lobbying and not lobbying. In that case, it must be strictly better off lobbying in state 1, and hence we have $\lambda^*(1) = 1$. Similarly, whenever $\lambda^*(1) \in (0,1)$ it must be that $\lambda^*(0) = 0$. This means, we cannot have a completely mixed symmetric equilibrium such that both $\lambda^*(1)$ and $\lambda^*(0)$ are in $(0,1)$.

An important implication of the above lemma is that in any symmetric equilibrium the interim beliefs of the PM must be such that $\beta^{Acc*}(0) < 1/2$ and $\beta^{Acc*}(1) \geq 1/2$. Hence, since $\beta^{Acc*}(1) = \pi/\delta^*$, we must have $\delta^* \in (0, 2\pi]$.

[33] Note that this requirement imposes no additional restriction if $\beta_j < 1$. It has bite only when $\beta_j = 1$. In that case the requirement is reasonable in the sense that if there is an arbitrarily small chance that the unaudited issue, i.e., issue j has state 0, then the PM would give priority to implementing reform on the audited issue, i.e., issue i for which he *knows* the state to be 1.

In order to characterize the symmetric equilibria, it is useful to define the following functions $z_n(\delta)$ and $\Gamma(\delta)$ over $\delta \in (0, 2\pi]$,

$$z_n(\delta) \equiv \binom{I-1}{n} \cdot \delta^n \cdot (1-\delta)^{I-1-n},$$

$$\Gamma(\delta) \equiv \sum_{n=0}^{K-1} z_n(\delta) + \sum_{n=K}^{I-1} \frac{K}{n+1} \cdot z_n(\delta)$$

and a function $\widetilde{\rho}(N, \delta)$ over $\delta \in (0, 2\pi]$ and $N \in \{K, \cdots, I\}$,

$$\widetilde{\rho}(N, \delta) \equiv \frac{1}{1-\Gamma(\delta)} \cdot \left\{ \sum_{n=K}^{N-1} \frac{n+1-K}{n+1} \cdot z_n(\delta) + \sum_{n=N}^{I-1} \frac{N - \min\{1, \pi/\delta\}K}{n+1} \cdot z_n(\delta) \right\}.$$

Assuming each IG lobbies with probability δ, $z_n(\delta)$ denotes the probability that n out of $I-1$ groups lobby, and $\Gamma(\delta)$ denotes the probability that a group that lobbies is granted access by the PM. To better understand this expression, note that it is comprised of two terms. The first term is the probability that there are less than K lobbying IGs, in which case an IG is granted access for sure. The second term adds up the probabilities associated with there being $n+1(> K)$ lobbying IGs, in which case each IG has $K/(n+1)$ probability of being granted access. Consider a candidate symmetric equilibrium of a lobbying game with agenda constraint N in which each IG lobbies with probability δ.[34] The term $\widetilde{\rho}(N, \delta)$ denotes the maximal probability that an IG that lobbies gets its issue reformed conditional on it not being granted access by the PM. To understand this expression, note that when there are fewer than N lobbying IGs, the maximal probability of getting reform (conditional on not being granted access) is 1, since $\beta^{Acc*}(0) \geq 1/2$. When there are $n+1(> N)$ lobbying IGs, the probability of an IG getting its issue reformed (conditional on not being granted access) depends on how many issues the PM granted access to but chose not to reform, i.e., on the expected number of "vacant" positions which is $N - \min\{1, \pi/\delta\}K$. Hence, each such IG has the maximal probability $N - \min\{1, \pi/\delta\}K/(n+1)$ of getting its issue reformed.

The following lemma provides necessary and sufficient conditions for the existence of the different types of symmetric lobbying equilibria.

LEMMA 4. *Consider a lobbying game with agenda constraint $N \in \{K, \cdots, I\}$. Define $\underline{f}(N)$ and $\overline{f}(N)$ as*

$$\begin{aligned} \underline{f}(N) &\equiv [1-\Gamma(\pi)] \cdot \widetilde{\rho}(\pi), \\ \overline{f}(N) &\equiv [1-\Gamma(\pi)] \cdot \widetilde{\rho}(\pi) + \Gamma(\pi). \end{aligned}$$

1. *A symmetric truthful lobbying equilibrium exists if and only if $f \in [\underline{f}(N), \overline{f}(N)]$.*
2. *A symmetric overlobbying equilibrium exists if $f \leq \underline{f}(N)$; a symmetric underlobbying equilibrium exists if $f \geq \overline{f}(N)$.*
3. *Moreover, if $[1-\Gamma(\delta)] \cdot \widetilde{\rho}(\delta) + \Gamma(\delta)$ is decreasing on $(0, 2\pi]$, then a symmetric overlobbying equilibrium exists only if $f < \overline{f}(N)$ and a symmetric underlobbying equilibrium exists only if $f > \overline{f}(N)$.*

[34] Note that given lemma 4, there is a unique δ associated with each symmetric equilibrium allowing us to use δ as a sufficient statistic for the lobbying strategies in a symmetric equilibrium.

To understand the term $\underline{f}(N)$, suppose that we are in a symmetric truthful lobbying equilibrium. Suppose an IG were to deviate in state 0 and choose to lobby. Since the PM believes each lobbying IG has state 1, it will get reform implemented with probability $[1-\Gamma(\pi)]\cdot\widetilde{\rho}(\pi)$, which is the probability that it is not granted access times the maximal probability of getting reform conditional on being not granted access. An IG in state 0 will not find it profitable to deviate if the lobbying cost, f, exceeds $\underline{f}(N)$. Similarly, $\overline{f}(N)$ $(\equiv [1-\Gamma(\pi)]\cdot\widetilde{\rho}(\pi)+\Gamma(\pi))$ is the payoff of an IG from lobbying in state 1 in a symmetric truthful lobbying equilibrium. An IG will not find it profitable to deviate to not lobbying in state 1 if the lobbying cost, f, is below $\overline{f}(N)$.

Furthermore, it can be shown that both $\underline{f}(N)$ and $\overline{f}(N)$ are strictly increasing in N with $\underline{f}(K)=0$ and $\overline{f}(I)=1$. Also, for any $N\in\{K,\cdots,I-1\}$, $[\underline{f}(N)\ ,\ \overline{f}(N)]\cap[\underline{f}(N+1)\ ,\ \overline{f}(N+1)]$ is non-empty. We can then state the following.

PROPOSITION 3. *For any lobbying cost $f\in(0,1)$, denote by $\mathcal{N}(f)\subseteq\{K,\cdots,I\}$ the set of agenda constraints for which a symmetric truthful lobbying equilibrium exists and let $\overline{N}(f)$ denote* $\max\mathcal{N}(f)$. *Then,*

1. *for any $f\in(0,1)$, $\mathcal{N}(f)$ is non-empty;*
2. *moreover, $\overline{N}(f)$ is increasing in f with* $\lim_{f\to 0}\overline{N}(f)=K$ *and* $\lim_{f\to 1}\overline{N}(f)=I$.

As with the two issues model in the previous section, introduction of agenda constraint yields benefits and costs. On the benefit side, as shown in the proposition above, there always exists an agenda constraint that induces full information revelation. On the cost side, agenda constraint reduces the PM's ability to implement welfare improving reform. Can there exist a Pareto improving agenda constraint? Our next example shows that it is indeed possible to introduce an optimal degree of agenda control that leads to Pareto improvement over no agenda constraint.

6.1.1. *Example*

Suppose there are three issues ($I=3$) and the PM can grant access to at most one IG ($K=1$); let $\pi=0.4$. We consider three possible levels of agenda constraint, with $N=1,2$ or 3; $N=3$ corresponding to the absence of an agenda constraint, $N=2$ corresponding to an agenda constraint that is less tight than the constraint on access and, finally, $N=1$ corresponding to an agenda constraint as tight as the access constraint. It can then be shown that

1. for $f\in(0,0.2928]$, $\mathcal{N}(f)=\{1\}$;
2. for $f\in[0.2928,0.3472]$, $\mathcal{N}(f)=\{1,2\}$;
3. for $f\in[0.3472,0.6528]$, $\mathcal{N}(f)=\{1,2,3\}$;
4. for $f\in[0.6528,9472]$, $\mathcal{N}(f)=\{2,3\}$;
5. for $f\in[0.9472,1]$, $\mathcal{N}(f)=\{3\}$.

Consider further, the case where $f=0.33$. For this cost, $N=1$ and 2 induce truthful lobbying in a symmetric equilibrium. Of these, $N=2$ yields higher payoffs to both the PM and the IGs. It can be shown that the payoff of the PM and the IGs in the truthful lobbying equilibrium are as follows: $EU(N=2)=2.936$ and $Ev(N=2)=0.10267$. However, what about $N=3$? When there is no agenda constraint, i.e., $N=3$, there exists a unique symmetric equilibrium which is an overlobbying equilibrium with $\delta^*=0.8$ and $\rho^*(1,\phi)=25/28\simeq 0.893$. The equilibrium payoffs are $EU(N=3)=2.104$ and $Ev(N=3)=0.0783$. Hence, we see that an optimal degree of agenda constraint ($N = 2$) leads to a strict Pareto improvement as compared to the absence of agenda constraint.

6.2. Subpoena power

We now go back to our baseline model with two IGs, and relax the assumption that the PM can grant access only to an IG that did lobby. Specifically, we award subpoena power to the PM, allowing him to grant access to any IG, whether this IG did lobby or not. This reflects a practice in several countries, where lawmakers can force people to produce documents or testify in legislative hearings. We are interested in determining whether, and how much, the PM can benefit from being awarded subpoena power. In the $N = 1$-game, the PM can already 'discipline' lobbying behavior through his access strategy. From now on, we shall therefore interest ourselves in the $N = 2$-game.

We start by providing a description of equilibria in the game where the PM has subpoena power.

LEMMA 5. *Consider the $N = 2-$game. Suppose that the PM has subpoena power.*

1. *There exists an equilibrium in which $\beta_i \in \{0, 1\}$ for each $i = 1, 2$ if and only if*
$$\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} \geq 1 \geq \frac{(1-\pi_1)\cdot f_1 + (1-\pi_2)\cdot f_2}{1-\pi_1\pi_2}$$
where β_i stands for the PM's posterior belief about θ_i.
2. *If $\frac{\pi_1 f_1+\pi_2 f_2}{\pi_1\pi_2} < 1$, the set of equilibria in which IGs lobby with positive probability is as follows:*
   (a) *The equilibrium strategies and beliefs in the game where the PM does not have subpoena power (case 2 of lemma 1) still constitute an equilibrium, with the only difference that the access strategy $\gamma_i(0,0) = 0$ for each $i = 1, 2$ is replaced with any $\gamma(0,0)$ satisfying $\sum_{i=1}^{2}\gamma_i(0,0) = 1$.*
   (b) *The following strategies and beliefs constitute an equilibrium if and only if $2\pi_1\alpha > 1$:*
$$\begin{cases} \lambda_1(1) = \frac{(2\pi_1\alpha-1)\cdot(2\alpha-1)}{4\alpha\cdot(\alpha-1)\cdot\pi_1} \text{ and } \lambda_1(0) = \frac{4\pi_1\alpha-1}{4\alpha\cdot(\alpha-1)\cdot(1-\pi_1)} \\ \lambda_2(1) = 1 \text{ and } \lambda_2(0) = \frac{\pi_2}{1-\pi_2} \\ 0 = \beta_2^{Acc}(0) < \beta_1^{Acc}(0) < \frac{1}{2} = \beta_2^{Acc}(1) < \beta_1^{Acc}(1) < 1 \\ \gamma_1(\ell_1, 0) = 1 \text{ and } \gamma_1(\ell_1, 1) = \left(1 - \frac{f_1}{2\pi_2}\right) \text{ for each } \ell_1 = 0, 1 \\ \rho_1(1,0) = 1 \text{ and } \rho_2(1,0) = \frac{2\pi_2 f_2}{2\pi_2 - f_1} \\ \rho_i(0,0) = 0 \text{ for each } i = 1, 2. \end{cases}$$

There is no other equilibrium in which IGs lobby with positive probability.

Thus, part (1) of lemma 5 identifies a condition which is necessary and sufficient for the existence of an equilibrium in which the PM gets perfectly informed about θ. Part (2) describes equilibria in the region of the parameter space where overlobbying occurs in the absence of subpoena power.

To go over the intuition underlying lemma 5 we partition the parameter space into three regions, and discuss each region in turn.

First, we consider the region of the parameter space where $\frac{(1-\pi_1)\cdot f_1+(1-\pi_2)\cdot f_2}{1-\pi_1\pi_2} > 1$. Observe that we then have $\frac{\pi_1 f_1+\pi_2 f_2}{\pi_1\pi_2} > 1$; thus, case 1 of lemma 1 applies.

This means that an equilibrium with truthful lobbying exists when the PM does not have subpoena power. In this case, the PM gets perfectly informed about θ. Part (1) of lemma 5 shows it is no longer true when the PM is awarded subpoena power. This is because in this region, lobbying is relatively costly. When the PM has subpoena power, IGs are then better off abstaining from lobbying and waiting for the PM to subpoena them. This is a strategy that IGs cannot follow when the PM does not have subpoena power. In this region of the parameter space, the PM is thus ex ante *strictly worse off* with subpoena power than without.

Second, we consider the region of the parameter space where the condition stated in part (1) of lemma 5 is satisfied. Observe that case 1 of lemma 1 applies again, meaning that, without subpoena power, the PM gets perfectly informed about θ through truthful lobbying decisions. Part (1) of lemma 5 shows that it is still the case when the PM has subpoena power. Thus, the PM is ex ante *as well off* with subpoena power as without. The difference with the region of the parameter space we have just discussed above is that lobbying is here less costly, up to the point where IGs are better off lobbying truthfully than hoping for the PM to subpoena them.

Third, we consider the region of the parameter space where $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$. Observe that case 2 of lemma 1, not case 1, now applies. This means that IGs overlobby in the unique equilibrium of the game without subpoena power, and that the PM gets only imperfectly informed about θ. This opens the possibility that the PM could gain from being awarded subpoena power.

Part (2a) of lemma 5 shows that the strategies[35] and beliefs of the unique equilibrium of the game without subpoena power still constitute an equilibrium of the game with subpoena power. Essentially, this happens because these lobbying strategies prescribe that an IG_i does not lobby only if $\theta_i = 0$, meaning that the decision to not lobby already reveals $\theta_i = 0$ and that there is no further information the PM can get by subpoenaing this IG.

Part (2b) of lemma 5 shows that this type of overlobbying equilibrium is no longer unique in the game with subpoena power. Specifically, a second type of equilibrium exists in the subregion of the parameter space where $2\pi_1\alpha > 1$. In these equilibria, IG_2 follows the same overlobbying strategy as in the equilibrium of the game without subpoena power. However, IG_1 lobbies with a smaller probability (resp. higher probability) when $\theta_1 = 1$ (resp. $\theta_1 = 0$). IG_1's lobbying decisions are therefore less informative than in the overlobbying equilibrium, inducing the PM to subpoena IG_1 with positive probability ($\gamma_1(0, \ell_2) > 0$) and forcing him to grant access to IG_2 with smaller probability. In order to prevent IG_2 from overlobbying more, the PM must then adopt $p_2 = 1$ with lower probability in those cases where IG_2 lobbies but is not granted access ($\rho_2(1, 0)$ is smaller than in the equilibrium of the game without subpoena power). Thus, the PM gets less information about θ than in the unique equilibrium of the game without subpoena power, making him ex ante *strictly worse off*.

To sum up, in neither of the three regions of the parameter space has the PM anything to gain from being awarded subpoena power. In other words, the PM should choose against being awarded subpoena power.

Our next proposition summarizes the above discussion. Before stating the proposition, we introduce some extra notation. Let EU^{sub} (resp. EU^{nosub}) denote

[35] With the exception of the PM's access strategy in the case where neither IG lobbies ($\gamma(0, 0)$).

the PM's highest equilibrium ex ante expected payoff in the game with subpoena power (resp. without subpoena power).

PROPOSITION 4. *Consider the $N = 2-$game. We have $EU^{sub} \leq EU^{nosub}$, with a strict inequality if and only if* $\frac{(1-\pi_1)\cdot f_1+(1-\pi_2)\cdot f_2}{1-\pi_1\pi_2} > 1$.

6.3. Endogenous interest groups

We go back to our baseline model where the PM does not have subpoena power. We now relax the assumption that the set of IGs is exogenously given. Specifically, we endogenize the set of IGs by adding to the game a preliminary stage at which IGs decide simultaneously whether to organize.[36] Once made, IGs' organization decisions are observed by all. If IG_i decides to organize, it bears a cost $c_i \geq 0$ and gets privately informed about θ_i. We are interested in determining the robustness of our results to endogenizing the set of IGs.[37]

Lemma 6 in the supplementary online appendix describes IGs' equilibrium organization strategies. Essentially, it establishes that IG_1's equilibrium organization strategy is independent of N. This happens because its issue is prioritized, implying that, in every subgame, its expected payoff is the same whether $N = 1$ or $N = 2$. By contrast, IG_2's organization strategy is shown to depend on N if and only if IG_1 organizes.[38] Specifically, when IG_1 organizes, IG_2 is (weakly) more likely (resp. less likely) to organize when $N = 1$ than when $N = 2$ if $\frac{\alpha\pi_1 f_1+(2\alpha-1)\cdot\pi_2 f_2}{\pi_1\pi_2} < 1$ (resp. $\frac{\alpha\pi_1 f_1+(2\alpha-1)\cdot\pi_2 f_2}{\pi_1\pi_2} > 1$).

The next proposition identifies the region of the parameter space where the introduction of an agenda constraint generates a Pareto improvement when the set of IGs is endogenous.[39]

PROPOSITION 5. *Suppose that the set of IGs is endogenous. We have $EU_k^{N=1} \geq EU_k^{N=2}$ for every player $k \in \{1, 2, PM\}$, with at least one inequality strict, if and only if each of the following three conditions holds:*

1. $c_1 \leq \pi_1 \cdot (1 - f_1)$;
2. $c_2 \leq \pi_2 \cdot (1 - \pi_1 - f_2)$; *and*
3. $\frac{\alpha\pi_1 f_1+(2\alpha-1)\cdot\pi_2 f_2}{\pi_1\pi_2} \leq 1$.

Compared to the case where the set of IGs is exogenous (proposition 2), two conditions are added which impose upper-bounds on the organization costs of the IGs.

The intuition runs as follows. For the introduction of an agenda constraint to generate a Pareto improvement, it must be that the PM gets better informed about θ when $N = 1$ than when $N = 2$. It follows that a Pareto-improving agenda constraint requires that:

[36] An IG's decision to organize can be interpreted in two ways. First, it can be interpreted as the IG's decision whether to open an office in D.C., Brussels or Canberra before knowing who is going to be the PM or whether the PM will benefit reforming the IG's issue. Alternatively, an organization decision can be interpreted as the IG's decision whether to acquire information about the realized state for its issue. Our preference goes for the first interpretation since it then makes more sense that each IG's organization decision is observed by the other players.

[37] or, alternatively, to endogenizing the information owned by IGs.

[38] If IG_1 does not organize, then IG_2 anticipates that, whether $N = 1$ or $N = 2$, it will be the only IG if it organizes, in which case the agenda constraint will not be binding (since the PM will not want to reform issue 1 given $\pi_1 < 1/2$). IG_2's expected payoff will then be independent of N.

[39] For expositional purposes, we assume that an IG chooses to organize whenever it is indifferent between organizing and not organizing.

1. IG_1 organizes (condition 1). This condition follows because, as we noted above, IG_1's equilibrium organization strategy is independent of N and IG_2's equilibrium organization strategy depends on N only if IG_1 organizes. Thus, if IG_1 were to not organize, each player's expected payoff would be independent of N, closing the door to the possibility of a Pareto improvement.
2. Either the IGs overlobby in the equilibrium of the $N = 2$-game or IG_2 is more likely to organize in the $N = 1$-game than in the $N = 2$-game (condition 3). This condition follows because, otherwise, the agenda constraint could not generate an informational gain for the PM.
3. IG_2 organizes in the $N = 1$-game (condition 2). This condition follows because, given conditions 1 and 3, IG_2's incentives to organize are stronger when $N = 1$ than when $N = 2$. In consequence, if IG_2 were to not organize when $N = 1$, it would not organize either when $N = 2$. IG_1 would then be the only organized IG, and each player's expected payoff would be independent of N, closing again the door to the possibility of a Pareto improvement.

To sum up, our result on Pareto-improving agenda constraint is robust to endogenizing the set of IGs, provided it is not too costly for IGs to organize.

7. CONCLUSION

We develop a model of informational lobbying and access in which a PM faces multiple issues on each of which he can implement a reform. A key feature of our model was that the PM faces resource constraints which inhibit his ability to provide access to IGs and may also restrict his ability to implement reform on all issues. We characterized the equilibrium of the lobbying game and showed that while the act of lobbying can signal pro-reform information, it may not do so perfectly. In particular, when the lobbying costs are not high, an IG may want to lobby the PM even when it does not possess pro-reform information in the hope that the PM is unable to verify the information provided. We then showed that the presence of agenda constraint can improve information transmission by making the disciplining role of access more credible. Indeed, in some cases imposition of an agenda constraint leads to Pareto improvement. Thus, we provide a new rationale for limiting the scope of decision making power of government (what the Economist (2014) article we quote called "Legislative hyperactivism"). The traditional understanding of such constraint is based not on information transmission but rather on wasteful rent seeking and bureaucratic red tape that new legislation inevitably brings. Our work, however, suggests that even in a set up where these sources of inefficiency are absent, the welfare of the decision maker as well as the lobbies can be improved by further constraining the decision maker.

We then extended our model to a more general case of several issues and showed that in such model one can find an optimal level of agenda constraint which leads to equilibrium in which the PM is fully informed. Second, we showed that in the baseline, two-issue model, giving PM subpoena power does not lead to improved information transmission. Finally, we also showed that our model is compatible with a more general model in which IGs are endogenously formed or information is endogenously acquired by IGs.

One can extend this line of research into several promising areas. One possibility is to endogenize the agenda and access constraints faced by the PM. For instance, one could allow the PM to optimally allocate his available time between access and policymaking. Another extension worth pursuing is to endogenize the precision of information. In our present model, an IG either chooses not to lobby, or to lobby and perfectly reveal the state of the world to the PM. One could allow lobbying to convey more or less precise signals about the state of the world, where the precision depends on the cost of lobbying. This will give us a richer framework to understand the extent to which IGs disclose information. One could also endogenize the lobbying cost incurred by the IGs via an all-pay auction. Here the magnitude of lobbying cost incurred could potentially signal the precision of available information. We leave these extensions for future work.

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8. APPENDIX

Proof of Lemma 1. We start by stating IG_i's lobbying problem. We denote the probability that IG_i lobbies by $\delta_i \equiv \pi_i \lambda_i(1) + (1-\pi_i)\lambda_i(0)$. We denote the probability that IG_i is granted access when it lobbies by $\Gamma_i(1) \equiv \delta_{-i}\gamma_i(1,1) + (1-\delta_{-i})$.

Given that the state for its issue is θ_i, IG_i chooses a lobbying strategy $\lambda_i(\theta_i)$ that solves

$$\max_{\lambda_i(\theta_i)} \quad Ev_i(\lambda_i(\theta_i))$$

where

$$Ev_i(\lambda_i(\theta_i)) = \lambda_i(\theta_i)[\Gamma_i(1)\rho_i(1,1) + (1-\Gamma_i(1))\rho_i(1,0) - f_i] + (1-\lambda_i(\theta_i))\rho_i(0,0).$$

We prove that an equilibrium in which $\lambda_i(1) = 1$ and $\lambda_i(0) = 0$ for $i = 1, 2$ exists if and only if $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} \geq 1$. To see this, let $\lambda_i(1) = 1$ and $\lambda_i(0) = 0$ for each $i = 1, 2$. It follows that $\beta_i(1,0) = 1$, $\beta_i(0,0) = 0$ and $\beta_i(1,1) = \theta_i$, implying $\rho_i(1,0) = 1$, $\rho_i(0,0) = 0$ and $\rho_i(1,1) = \theta_i$. Also, $\delta_i = \pi_i$ and $\Gamma_i(1) = \pi_{-i}\gamma_i(1,1) + (1-\pi_{-i})$.

The necessity part follows because $\lambda_i(0) = 0$ requires

$$\frac{dEv_i}{d\lambda_i(0)} \leq 0 \Leftrightarrow 1 - \Gamma_i(1) - f_i \leq 0.$$

We then get that

$$\begin{cases} \frac{dEv_1}{d\lambda_1(0)} \leq 0 \Leftrightarrow \gamma_1(1,1) \geq 1 - \frac{f_1}{\pi_2} \\ \frac{dEv_2}{d\lambda_2(0)} \leq 0 \Leftrightarrow \gamma_1(1,1) \leq \frac{f_2}{\pi_1}. \end{cases}$$

Hence $\gamma_1(1,1) \in \left[1 - \frac{f_1}{\pi_2}, \frac{f_2}{\pi_1}\right]$. The latter interval is non-empty if and only if $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} \geq 1$.

The sufficiency part follows straightforwardly by setting $\gamma_1(1,1) \in \left[1 - \frac{f_1}{\pi_2}, \frac{f_2}{\pi_1}\right]$ (which is possible given that (i) $X_i(\rho) = 0$ for each $i = 1, 2$, and (ii) the interval is non-empty since $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} \geq 1$) and observing that $\frac{dEv_i}{d\lambda_i(1)} = 1 - f_i > 0$ (which implies $\lambda_i(1) = 1$).

We prove part (2) of the statement. Suppose that $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$. We proceed in several steps.

First, we establish that $\beta_i^{Acc}(0) < 1/2$, implying $\rho_i(0,0) = 0$ for each $i = 1, 2$. We proceed by contradiction. Assume that $\beta_i^{Acc}(0) \geq 1/2$ for some i. Since $\pi_i < 1/2$, it must be that either $\lambda_i(0) > \lambda_i(1)$ or $\lambda_i(1) = \lambda_i(0) = 1$. In either case, $\lambda_i(0) \geq \lambda_i(1)$ and $\pi_i < 1/2$ imply $\beta_i^{Acc}(1) < 1/2$ and, therefore, $\rho_i(1,0) = 0$. It follows that $\frac{dEv_i}{d\lambda_i(0)} = -f_i - \rho_i(0,0) < 0$, which contradicts $\lambda_i(0) > 0$.

Second, we establish that $\delta_i > 0$ for each $i = 1, 2$. We proceed by contradiction. Assume that $\lambda_i(1) = \lambda_i(0) = 0$ for some i. It follows that $\beta_i^{Acc}(0) = \pi_i < 1/2$, which implies $\rho_i(0,0) = 0$. Moreover, we have $\Gamma_{-i}(1) = 1$, implying $\lambda_{-i}(1) = 1$ and $\lambda_{-i}(0) = 0$. It follows that $\delta_{-i} = \pi_{-i}$, $\Gamma_i(1) \geq 1 - \pi_{-i}$ and $X_{-i}(\rho) = 0$. At the same time, $\lambda_i(1) = 0$ requires

$$\frac{dEv_i}{d\lambda_i(1)} \leq 0 \Leftrightarrow \Gamma_i(1) + (1-\Gamma_i(1))\rho_i(1,0) \leq f_i.$$

There are three possible cases.

1. $\beta_i^{Acc}(1) \in (0,1)$, which implies $X_i(\rho) > 0$. Since $X_{-i}(\rho) = 0$, we get $\Gamma_i(1) = 1$. It follows that $\frac{dEv_i}{d\lambda_i(1)} = 1 - f_i > 0$.
2. $\beta_i^{Acc}(1) = 1$, which implies $\rho_i(1,0) = 1$. We get again $\frac{dEv_i}{d\lambda_i(1)} = 1 - f_i > 0$.
3. $\beta_i^{Acc}(1) = 0$, which implies $\rho_i(1,0) = 0$. Since $\Gamma_i(1) \geq 1 - \pi_{-i}$, we get $\frac{dEU_i}{d\lambda_i(R)} \geq 1 - \pi_i - f_i > 0$, the strict inequality since $\pi_{-i} < 1/2$ and $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$. It follows that $\frac{dEv_i}{d\lambda_i(1)} > 0$.

In all three cases, $\frac{dEv_i}{d\lambda_i(1)} > 0$ contradicts $\lambda_i(1) = 0$.

Third, we establish that $\beta_i^{Acc}(1) \geq 1/2$, which requires $\lambda_i(1) > \lambda_i(0)$ for each $i = 1, 2$. Assume by way of contradiction that $\beta_i^{Acc}(1) < 1/2$ for some i. It follows that $\rho_i(1,0) = 0$ and, therefore, that

$$\frac{dEv_i}{d\lambda_i(0)} = -f_i - \rho_i(0,0) < 0.$$

The strict inequality implies $\lambda_i(0) = 0$. Since we have shown above that $\delta_i > 0$, we must then have $\lambda_i(1) > 0$. Together $\lambda_i(1) > 0$ and $\lambda_i(0) = 0$ imply $\beta_i^{Acc}(1) = 1$, a contradiction. Hence $\beta_i^{Acc}(1) \geq 1/2$ for each $i = 1, 2$. Given $\delta_i > 0$ and $\pi_i < 1/2$, $\beta_i^{Acc}(1) \geq 1/2$ requires $\lambda_i(1) > \lambda_i(0)$.

Fourth, we establish $[\lambda_i(1) - \lambda_i(0)] < 1$ for each $i = 1, 2$. Assume by way of contradiction that $\lambda_i(1) = 1$ and $\lambda_i(0) = 0$ for some i. It follows that $X_i(\rho) = 0$. Moreover, $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$ implies $[\lambda_{-i}(1) - \lambda_{-i}(0)] < 1$. Two cases are possible:

1. $\lambda_{-i}(0) = 0$ and $\lambda_{-i}(1) \in (0,1)$. In this case, we have $\beta_{-i}^{Acc}(1) = 1$, which implies $\rho_{-i}(1,0) = 1$. It follows that $\frac{dEv_{-i}}{d\lambda_{-i}(1)} = 1 - f_{-i} > 0$, contradicting $\lambda_{-i}(1) < 1$.
2. $\lambda_{-i}(0) > 0$. In this case, we have $\beta_{-i}^{Acc}(1) < 1$, which implies $X_{-i}(\rho) > 0$. Since $X_i(\rho) = 0$, we get $\Gamma_{-i}(1) = 1$ and $\frac{dEv_{-i}}{d\lambda_{-i}(0)} = -f_{-i} < 0$, contradicting $\lambda_{-i}(0) = 0$.

Since these two cases exhaust all possibilities, we have established $[\lambda_i(1) - \lambda_i(0)] < 1$ for each $i = 1, 2$.

Fifth, we establish $\beta_1^{Acc}(1) > 1/2$ and $\beta_2^{Acc}(1) = 1/2$.

We start by establishing that $\beta_i^{Acc}(1) = 1/2$ for some i. Assume by way of contradiction that $\beta_i^{Acc}(1) > 1/2$ for each $i = 1, 2$. It follows that $\rho_i(1,0) = 1$, implying $\lambda_i(1) = 1$ and $\lambda_i(0) \in (0,1)$. The latter requires

$$\frac{dEv_i}{d\lambda_i(0)} = 0 \Leftrightarrow 1 - \Gamma_i(1) = f_i.$$

Moreover, we get $\delta_i > \pi_i$. From the condition that $1 - \Gamma_i(1) = f_i$ for each $i = 1, 2$, we get

$$\Gamma_1(1) + f_1 = \Gamma_2(1) + f_2 \Rightarrow \gamma_1(1,1) = \frac{\delta_2 + (f_2 - f_1)}{\delta_1 + \delta_2}.$$

At the same time, $1 - \Gamma_2(1) = f_2$ implies $\gamma_1(1,1) = f_2/\delta_1$. Taken together the two expressions for $\gamma_1(1,1)$ imply $\frac{\delta_1 f_1 + \delta_2 f_2}{\delta_1 \delta_2} = 1$. The contradiction follows since $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$ and $\delta_i > \pi_i$ for each $i = 1, 2$. Hence $\beta_i^{Acc}(1) = 1/2$ for some i.

We continue by establishing $\beta_2^{Acc}(1) = 1/2$. Assume by way of contradiction that $\beta_2^{Acc}(1) > 1/2$. It follows from above that $\beta_1^{Acc}(1) = 1/2$, implying $X_1(\rho) = \frac{1}{2} > X_2(\rho)$. Since $\alpha > 1$, we get $\Gamma_1(1) = 1$, implying $\frac{dEv_1}{d\lambda_1(0)} = -f_1 < 0$. It follows that $\lambda_1(0) = 0$ which, together with $\delta_1 > 0$, implies $\lambda_1(1) > 0$ and $\beta_1^{Acc}(1) = 1$, a contradiction. Hence $\beta_2^{Acc}(1) = 1/2$, implying $X_2(\rho) = 1/2$.

It remains to establish $\beta_1^{Acc}(1) > 1/2$ and $\rho_1(1,0) = 1$. Assume by way of contradiction that $\beta_1^{Acc}(1) = 1/2$. It follows that $X_1(\rho) = 1/2$. Since $X_2(\rho) = 1/2$ and $\alpha > 1$, we get $\Gamma_1(1) = 1$, implying $\frac{dEv_1}{d\lambda_1(0)} = -f_1 < 0$ and, therefore, $\lambda_1(0) = 0$. This, together with $\delta_1 > 0$, implies $\lambda_1(1) > 0$ and $\beta_1^{Acc}(1) = 1$, a contradiction. Hence $\beta_1^{Acc}(1) > 1/2$, implying $\rho_1(1,0) = 1$.

Sixth, we establish $\lambda_i(1) = 1$ for each $i = 1, 2$. This is obvious for IG_1 since $\rho_1(1,0) = 1$ implies $\frac{dEv_1}{d\lambda_1(1)} = 1 - f_1 > 0$. It remains to consider IG_2. Assume by way of contradiction that $\lambda_2(R) \in (0,1)$. Observe that $\beta_2^{Acc}(1) = 1/2$, together with $\delta_2 > 0$ and $\pi_2 < 1/2$, implies $\lambda_2(0) \in (0,1)$. Given $\lambda_2(\theta_2) \in (0,1)$ for each θ_2, it must be that

$$\begin{cases} \frac{dEv_2}{d\lambda_2(1)} = 0 \Leftrightarrow \Gamma_2(1) + (1 - \Gamma_2(1))\,\rho_2(1,0) = f_2 \\ \frac{dEv_2}{d\lambda_2(0)} = 0 \Leftrightarrow (1 - \Gamma_2(1))\,\rho_2(1,0) = f_2. \end{cases}$$

These two equalities can be satisfied simultaneously only if $\Gamma_2(1) = 0$, which is possible only if $\delta_1 = 1$, contradicting $\lambda_1(1) > \lambda_1(0)$.

Seventh, we determine $\lambda_i(0)$ for each $i = 1, 2$. Observe that $\beta_i^{Acc}(1) = \pi_i / [\pi_i + (1 - \pi_i)\,\lambda_i(0)]$. For IG_2, we have $\beta_i^{Acc}(1) = 1/2$, which implies $\lambda_i(0) = \pi_i / (1 - \pi_i) \in (0,1)$. For IG_1, we have $\beta_1^{Acc}(1) > 1/2$, which implies $\rho_1(1,0) = 1$. Together $[\lambda_1(1) - \lambda_1(0)] < 1$, $\lambda_1(1) = 1$ and $\lambda_1(1) > \lambda_1(0)$ imply $\lambda_1(0) \in (0,1)$. For $\lambda_i(0) \in (0,1)$ for each $i = 1, 2$, it must be that $\gamma_i(1,1) \in (0,1)$ and, therefore, that $X_1(\rho)\,\alpha = X_2(\rho)$. Since $X_1(\rho) = 1 - \beta_1^{Acc}(1)$ and $X_2(\rho) = 1/2$, this equality implies $\lambda_1(0) = \frac{\pi_1}{1-\pi_1}\frac{1}{2\alpha - 1} \in (0,1)$.

Eight, and last, it remains to determine $\gamma_i(1,1)$ and $\rho_i(1,0)$ for each $i = 1, 2$. Recall that the expected probability that IG_i is granted access if it lobbies is given by $\Gamma_i(1) \equiv \delta_{-i}\gamma_i(1,1) + (1 - \delta_{-i})$. Given $\lambda_i(1) = 1$ and $\lambda_{-i}(0) = \frac{\pi_{-i}}{1-\pi_{-i}}\frac{1}{2\alpha_{-i}-1}$ (where $\alpha_1 = \alpha$ and $\alpha_2 = 1$), we have $\delta_{-i} = \frac{2\pi_{-i}}{2 - \frac{1}{\alpha_{-i}}}$. Moreover, since $\lambda_i(0) \in (0,1)$ for each $i = 1, 2$, it must be that

$$\frac{dEv_i}{d\lambda_i(0)} = 0 \Leftrightarrow (1 - \Gamma_i(1))\,\rho_i(1,0) = f_i \quad \text{for each } i = 1, 2.$$

Plugging the above expressions in this equality, we get

$$\gamma_{-i}(1,1)\,\rho_i(1,0) = \frac{2 - \frac{1}{\alpha_{-i}}}{2\pi_{-i}} f_i \quad \text{for each } i = 1, 2.$$

We know from step 5 that $\rho_1(1,0) = 1$. It follows that the above equality for $i = 1$ yields $\gamma_2(1,1) = \frac{f_1}{2\pi_2} \in (0,1)$. In turn, and knowing that $\gamma_1(1,1) = 1 - \gamma_2(1,1)$, the above equality for $i = 2$ yields $\rho_2(1,0) = \frac{\pi_2(2\alpha-1)f_2}{\pi_1\alpha(2\pi_2 - f_1)} \in (0,1)$. ■

Proof of Lemma 2. We start by stating IG_i's lobbying problem. Given θ_i, IG_i chooses a lobbying strategy $\lambda_i(\theta_i)$ that solves

$$\max_{\lambda_i(\theta_i) \in [0,1]} Ev_i(\lambda_i(\theta_i))$$

where

$$\begin{aligned} Ev_i(\lambda_i(\theta_i)) = {} & \lambda_i(\theta_i)\,[\delta_{-i}\gamma_i\rho_i(1,1;1,0) + (1-\gamma_i)\,[\pi_{-i}\lambda_{-i}(1)\,\rho_i(1,1;0,1) \\ & + (1-\pi_{-i})\,\lambda_{-i}(0)\,\rho_i(1,1;0,1)] + (1-\delta_{-i})\,\rho_i(1,0;1,0) - f_i] \\ & + (1-\lambda_i(\theta_i))\,[\pi_{-i}\lambda_{-i}(1)\,\rho_i(0,1;0,1) + (1-\pi_{-i})\,\lambda_{-i}(0)\,\rho_i(0,1;0,1) + (1-\delta_{-i})\,\rho_i(0,0;0,0)], \end{aligned}$$

with $\rho_i(\ell_i, \ell_{-i}; a_i, a_{-i})$ and γ_i as a shorthand for $\gamma_i(1,1)$.

We prove that when $f_2 > (1-\pi_1)$, an equilibrium exists in which $\lambda_1(1) = 1$ and $\lambda_1(0) = \lambda_2(1) = \lambda_2(0) = 0$. In this case, $\delta_1 = \pi_1$ and $\delta_2 = 0$. Moreover, $\beta_1^{Acc}(1) = 1$, $\beta_1^{Acc}(0) = 0$ and $\beta_2^{Acc}(0) = \pi_2$. We can infer from these beliefs that

$$\begin{cases} \rho_1(1,1;0,1) = 1 \\ \rho_1(0,\cdot;0,\cdot) = \rho_2(0,\cdot;0,\cdot) = \rho_2(1,1;1,0) = 0 \\ \rho_1(1,\cdot;1,0) = \theta_1 \\ \rho_2(1,0;1,0) = \theta_2 \text{ and } \rho_2(1,1;0,1) = 1-\theta_1. \end{cases}$$

Let $\beta_2^{Acc}(1)$ take any value in $[0,1]$. Also, let $\rho_2(1,1;0,1) \in [0,1]$ be consistent with $\beta_2^{Acc}(1)$. It follows that $X_1(\rho)\alpha = X_2(\rho) = 0$ (since IG$_1$ lobbies if and only if $\theta_1 = 1$), and γ_i can take any value in $[0,1]$ (with the restriction that $\gamma_1 + \gamma_2 = 1$). We then get

$$\begin{cases} \frac{dEv_1}{d\lambda_1(1)} = 1 - f_1 > 0 \text{ and } \frac{dEv_1}{d\lambda_1(0)} = -f_1 < 0 \\ \frac{dEv_2}{d\lambda_2(1)} = (1-\pi_1) - f_2 < 0 \text{ and } \frac{dEv_2}{d\lambda_2(0)} = -f_2 < 0, \end{cases}$$

which is consistent with the lobbying strategies.

We continue by proving that when $f_2 > 1 - \pi_1$, in any equilibrium we have $\lambda_i(1) = 1$ and $\lambda_i(0) = \lambda_{-i}(1) = \lambda_{-i}(0) = 0$ for some $i \in \{1,2\}$.

First, we observe that $\lambda_i(1) \geq \lambda_i(0)$ and $\lambda_i(0) < 1$ for each $i = 1,2$. To see this, assume by way of contradiction that $\lambda_i(0) \in (\lambda_i(1), 1]$. Observe that this implies $\lambda_i(0) > 0$. Moreover, we get $\beta_i^{Acc}(1) < 1/2$, implying $\frac{dEv_i}{d\lambda_i(0)} = -f_i < 0$ and, therefore, contradicting $\lambda_i(0) > 0$.

Second, we establish that together $\lambda_1(1) = 1$ and $\lambda_1(0) = 0$ imply $\lambda_2(1) = \lambda_2(0) = 0$. Given $\lambda_1(1) = 1$ and $\lambda_1(0) = 0$, we have $\delta_1 = \pi_1$. Moreover, $\beta_1^{Acc}(1) = 1$ and $\beta_1^{Acc}(0) = 0$ which, together with $\alpha > 1$, imply $\rho_1(1,1;0,1) = 1$. It follows that

$$\frac{dEv_2}{d\lambda_2(0)} \leq \frac{dEv_2}{d\lambda_2(1)} = (1-\pi_1) - f_2 - (1-\pi_1)\rho_2(0,0;0,0) < 0,$$

which implies $\lambda_2(1) = \lambda_2(0) = 0$.

Third, we establish that $\lambda_1(1) < 1$ and/or $\lambda_1(0) > 0$ implies $\lambda_1(1) = \lambda_1(0) = 0$, $\lambda_2(1) = 1$ and $\lambda_2(0) = 0$. There are three cases to consider:

1. $\underline{\lambda_1(1) = 1}$. In this case, $\lambda_1(0) \in (0,1)$ and $\delta_1 < 1$. It follows that $\beta_1^{Acc}(1) < 1$. Moreover, $\beta_1^{Acc}(0) = 0$, which implies $\rho_1(0,\cdot;0,\cdot) = 0$ and $\rho_2(1,0;1,0) = \theta_2$. We then get

$$\begin{cases} \frac{dEv_2}{d\lambda_2(1)} = \delta_1\gamma_2\rho_2(1,1;1,0) + (1-\gamma_2)(1-\pi_1)\lambda_1(0)\rho_2(1,1;0,1) + (1-\delta_1) - f_2 \\ \frac{dEv_2}{d\lambda_2(0)} = (1-\gamma_2)(1-\pi_1)\lambda_1(0)\rho_2(1,1;0,1) - f_2. \end{cases}$$

   Given $f_2 > (1-\pi_1)$, we have $\frac{dEv_2}{d\lambda_2(0)} < 0$ and, therefore, $\lambda_2(0) = 0$. It must then be that $\lambda_2(1) > 0$ since otherwise $\frac{dEv_1}{d\lambda_1(0)} = -f_1 < 0$, which would contradict $\lambda_1(0) > 0$. But $\lambda_2(1) > 0 = \lambda_2(0)$ implies that $\beta_2^{Acc}(1) = 1$ and, together with $\beta_1^{Acc}(1) < 1$, that $X_2(\rho) < X_1(\rho)\alpha$. But then $\gamma_1 = 1$ and, again $\frac{dEv_1}{d\lambda_1(0)} = -f_1 < 0$, contradicting $\lambda_1(0) > 0$.
2. $\underline{\lambda_1(1) \in (0,1)}$. We first observe that $\lambda_1(0) = 0$. We already know that $\lambda_1(0) < 1$. Assume by way of contradiction that $\lambda_1(0) \in (0,1)$. It must then be that $\frac{dEv_1}{d\lambda_1(1)} = \frac{dEv_1}{d\lambda_1(0)} = 0$, or $\delta_2\gamma_2 = 1$. The latter requires $\delta_2 = 1$, which we

already know cannot be possible. Hence $\lambda_1(0) = 0$. Given $\lambda_1(1) > \lambda_1(0) = 0$ and $\pi_1 < 1/2$, we get $\beta_1^{Acc}(1) = 1$ and $\beta_1^{Acc}(0) < 1/2$ and, therefore, $\rho_1(1,1;0,1) = 1$ and $\rho_1(0,\cdot;0,\cdot) = 0$. It follows that $\frac{dEv_1}{d\lambda_1(1)} = 1 - f_1 > 0$, which contradicts $\lambda_1(1) < 1$.

3. $\underline{\lambda_1(1) = 0.}$ Since $\lambda_1(1) \geq \lambda_1(0)$, we then have $\lambda_1(0) = 0$. It follows that $\beta_1^{Acc}(0) = \pi_1 < 1/2$ and, therefore, $\rho_1(0,\cdot;0,\cdot) = 0$. Moreover, $\frac{dEv_2}{d\lambda_2(0)} \leq -f_2 < 0$, which implies $\lambda_2(0) = 0$. It follows that $\beta_2^{Acc}(0) < 1/2$ and, therefore, $\rho_2(0,\cdot;0,\cdot) = 0$. We then get $\frac{dEv_2}{d\lambda_2(1)} = 1 - f_2 > 0$, which implies $\lambda_2(1) = 1$.

We now prove that when $f_2 \leq 1 - \pi_1$, an equilibrium exists in which $\lambda_i(1) = 1$ and $\lambda_i(0) = 0$ for each $i = 1,2$. In this case, $\delta_i = \pi_i$. Moreover, $\beta_i^{Acc}(1) = 1$ and $\beta_i^{Acc}(0) = 0$, from which it follows that:

1. $X_1(\rho)\alpha = X_2(\rho)$, implying we can let $\gamma_1 = (1 - \gamma_2) \in \left(1 - \frac{f_1}{\pi_2}, 1\right]$;
2. $\rho_i(1,1;0,1) = 1$ if $\theta_{-i} = 0$,
3. $\rho_i(0,\cdot;0,\cdot) = 0$; and
4. $\rho_1(1,1;1,0) = \theta_1$ and $\rho_1(1,1;0,1) = 1$.

It follows that $\frac{dEv_i}{d\lambda_i(1)} \geq 0$ and $\frac{dEv_i}{d\lambda_i(0)} < 0$ for each $i = 1,2$, which is consistent with $\lambda_i(1) = 1$ and $\lambda_i(0) = 0$. The construction of the equilibrium is completed by letting the remaining access and policy choice strategies be as specified in section 3.

It remains to prove that when $f_i < 1 - \pi_i$ for each $i = 1,2$, in any equilibrium we have $\lambda_i(1) = 1$ and $\lambda_i(0) = 0$ for each $i = 1,2$. We proceed via a sequence of seven steps.

First, we show that $\beta_i^{Acc}(0) < 1/2$ for each $i = 1,2$. Assume by way of contradiction that $\beta_i^{Acc}(0) \geq 1/2$ for some $i \in \{1,2\}$. This is possible only if either $\lambda_i(1) < \lambda_i(0)$ or $\lambda_i(1) = \lambda_i(0) = 1$. In either case, $\beta_i^{Acc}(1) < 1/2$, implying $\frac{dEv_i}{d\lambda_i(0)} \leq -f_i < 0$ and, therefore, contradicting $\lambda_i(0) > 0$.

Second, we show that $\lambda_i(1) \geq \lambda_i(0)$ for each $i = 1,2$. Assume by way of contradiction that $\lambda_i(1) < \lambda_i(0)$ for some $i \in \{1,2\}$. This implies $\beta_i^{Acc}(1) < 1/2$. We get a contradiction following the same argument as in the first step.

Third, we show that $\lambda_i(0) < 1$ for each $i = 1,2$. Assume by way of contradiction that $\lambda_i(0) = 1$. Since we already know that $\lambda_i(1) \geq \lambda_i(0)$, we have $\lambda_i(1) = \lambda_i(0) = 1$ and, therefore, $\beta_i^{Acc}(1) = \pi_i < 1/2$. Again, we get a contradiction following the same argument as in the first step.

Fourth, we show that $\lambda_i(1) > 0$ for each $i = 1,2$. Assume by way of contradiction that $\lambda_i(1) = 0$ for some $i \in \{1,2\}$. Since we already know that $\lambda_i(1) \geq \lambda_i(0)$, we have $\lambda_i(1) = \lambda_i(0) = 0$ and, therefore, $\delta_i = 0$. Since $\beta_i^{Acc}(0) < 1/2$, we then get $\lambda_{-i}(1) = 1$ and $\lambda_{-i}(0) = 0$. But then $\frac{dEv_i}{d\lambda_i(1)} \geq (1 - \pi_i) - f_i > 0$, which contradicts $\lambda_i(1) = 0$.

Fifth, we show that $\lambda_i(1) > \lambda_i(0)$ for each $i = 1,2$. We already know from the second step that $\lambda_i(1) \geq \lambda_i(0)$ for each $i = 1,2$. Assume by way of contradiction that $\lambda_i(1) = \lambda_i(0)$ for some $i \in \{1,2\}$. Also, we know from the fourth step that $\lambda_i(1) > 0$. It follows that $\beta_i^{Acc}(1) = \pi_i < 1/2$, and the same argument as in the first step applies. Observe that this step implies $\delta_i \in (0,1)$ for each $i = 1,2$.

Sixth, we show that $\lambda_i(1) = 1$ for each $i = 1,2$. Assume by way of contradiction that $\lambda_i(1) < 1$ for some i. We already know from the fifth step that $\lambda_i(1) > \lambda_i(0)$.

It follows that $\lambda_i(1) \in (0,1)$ and, therefore, that $\frac{dEv_i}{d\lambda_i(1)} = 0$. The latter, together with $\delta_{-i} < 1$, implies that $\frac{dEv_i}{d\lambda_i(0)} < 0$ and, therefore, that $\lambda_i(0) = 0$ and $\beta_i^{Acc}(1) = 1$. Also, we have $\frac{dEv_i}{d\lambda_i(1)} = 0$ only if $(1 - \delta_{-i}) \leq f_i$ which, together with $f_i < 1 - \pi_{-i}$, requires $\lambda_{-i}(0) > 0$. It follows that $\beta_{-i}^{Acc}(1) < 1$.

If $i = 1$, $\beta_1^{Acc}(1) = 1$ and $\alpha > 1$ imply $\frac{dEU_1}{d\lambda_1(1)} = 1 - f_1 > 0$, which contradicts $\lambda_1(1) < 1$.

If $i = 2$, $\lambda_1(0) > 0$ requires $\gamma_1 < 1$ and, therefore, $X_1(\rho)\alpha \leq X_2(\rho)$. Simple algebra shows that, together, $\beta_2^{Acc}(1) = 1$ and $\lambda_1(0) > 0$ imply $X_1(\rho)\alpha > X_2(\rho)$, a contradiction.

Seventh, and last, we show that $\lambda_i(0) = 0$ for each $i = 1,2$. Assume by way of contradiction that $\lambda_i(0) > 0$ for some i. We already know from the fifth step that $\lambda_i(1) > \lambda_i(0)$, which implies $\lambda_i(0) \in (0,1)$. Moreover, given $\lambda_{-i}(1) = 1$, $\lambda_i(0) > 0$ requires $\lambda_{-i}(0) \in (0,1)$ as well. Finally, for $\lambda_i(0) \in (0,1)$ for each $i = 1,2$, it must be that $\gamma_i \in (0,1)$, which requires $X_1(\rho)\alpha = X_2(\rho)$. Simple, but tedious, algebra shows that $X_1(\rho) > 0$ and $X_1(\rho) \geq X_2(\rho)$ which, together with $\alpha > 1$, contradicts $X_1(\rho)\alpha = X_2(\rho)$. ∎

Proof of Proposition 1. The result follows directly from lemmata 1 and 2.

Consider first the case where $f_2 > 1 - \pi_1$. Simple algebra establishes that $f_2 > 1 - \pi_1$ implies $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} > 1$. From lemma 1 we know that equilibrium lobbying is truthful in the game without agenda constraint. Hence, $\beta_i^{N=2} \in \{0,1\}$ for each $i = 1,2$. From lemma 2 we know that in the game with agenda constraint, IG_1 lobbies truthfully while IG_2 does not lobby. Hence, $\beta_1^{N=1} \in \{0,1\}$ and $\beta_2^{N=1} = \pi_2$.

Consider second the case where $f_2 \in \left[\pi_1\left(1 - \frac{f_1}{\pi_2}\right), 1 - \pi_1\right]$. In other words, $f_2 \leq 1 - \pi_1$ and $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} \geq 1$. For the game without agenda constraint, equilibrium lobbying and posterior beliefs are thus as described in the previous case. For the game with agenda constraint, we know from lemma 2 that equilibrium lobbying is truthful. It follows that $\beta_i^{N=1} \in \{0,1\}$ for each $i = 1,2$.

Consider third the case where $f_2 < \pi_1\left(1 - \frac{f_1}{\pi_2}\right)$. In other words, $f_2 < 1 - \pi_1$ and $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$. For the game with agenda constraint, equilibrium lobbying and posterior beliefs are thus as described in the previous case. For the game without agenda constraint, we know from lemma 1 that, in equilibrium, IGs overlobby. It follows that $\beta_i^{N=2}(1,0) \in [1/2, 1)$ for each $i = 1,2$. ∎

Proof of Proposition 2. We start by establishing the sufficiency of the condition in the statement of proposition 2. Observe that this condition, together with $\alpha > 1$, implies $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$ and $f_2 < 1 - \pi_1$. Thus, case 2 of lemma 1 and case 2 of lemma 2 apply.

In the $N = 1$-game, equilibrium lobbying is truthful and the PM prioritizes issue 1. The PM chooses $p = (1,0)$ when $\theta = (1,1)$. He chooses $p = \theta$ otherwise. Players' expected payoffs are then given by

$$\begin{aligned} EU_1^{N=1} &= \pi_1(1 - f_1) \\ EU_2^{N=1} &= \pi_2(1 - \pi_1 - f_2) \\ EU_{PM}^{N=1} &= \alpha + (1 - \pi_1 \pi_2). \end{aligned}$$

In the $N = 2$-game, IGs overlobby in equilibrium. Given the strategies and beliefs described in the statement and the proof of lemma 1, we get that players' expected payoffs are given by

$$\begin{aligned}
EU_1^{N=2} &= \pi_1 (1 - f_1) \\
EU_2^{N=2} &= \pi_2 \left[1 - \delta_1 \gamma_1 (1,1) (1 - \rho_2 (1,0)) - f_2\right] \\
&= \pi_2 \left\{ 1 - \pi_1 \left[\frac{\alpha (2\pi_2 - f_1) - \frac{\pi_2}{\pi_1} (2\alpha - 1) f_2}{\pi_2 (2\alpha - 1)} \right] - f_2 \right\} \\
EU_{PM}^{N=2} &= (\alpha + 1) - \frac{2\pi_1 \pi_2 \alpha}{2\alpha - 1}.
\end{aligned}$$

Thus, $EU_1^{N=1} = EU_1^{N=2}$ and $EU_{PM}^{N=1} > EU_{PM}^{N=2}$. Moreover, the condition in the statement of proposition 2 implies $EU_2^{N=1} \geq EU_2^{N=2}$.

We now establish the necessity of the condition in the statement of proposition 2. Suppose $EU_k^{N=1} \geq EU_k^{N=2}$ for every player $k \in \{1, 2, PM\}$, with at least one inequality strict.

We start by observing that we must have $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$. To see this, assume the contrary. Case 1 of lemma 1 would then apply. In the $N = 2$-game, equilibrium lobbying would be truthful. IG_2's equilibrium expected payoff would thus be given by

$$EU_2^{N=2} = \pi_2 (1 - f_2).$$

In the $N = 1$-game, IG_2 would either lobby truthfully (if $f_2 \leq 1 - \pi_1$) or would abstain from lobbying (if $f_2 > 1 - \pi_1$). IG_2's equilibrium expected payoff would thus be given by

$$EU_2^{N=1} = \begin{cases} \pi_2 (1 - \pi_1 - f_2) & \text{if } f_2 \leq 1 - \pi_1 \\ 0 & \text{if } f_2 > 1 - \pi_1. \end{cases}$$

Simple algebra establishes $EU_2^{N=2} > EU_2^{N=1}$, a contradiction.

We continue by observing that $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$ implies case 2 of lemma 1 and case 2 of lemma 2 apply. As shown in the proof of sufficiency above, we have in this case that $EU_2^{N=1} \geq EU_2^{N=2}$ if and only if $\frac{\alpha \pi_1 f_1 + (2\alpha - 1) \pi_2 f_2}{\pi_1 \pi_2} \leq 1$. Hence the necessity of this condition. Observe that this condition implies that $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$ is satisfied. ■