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Structural analysis of first-price auction data: insights from the laboratory *

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Abstract

We use laboratory data from first-price auctions and manipulate the *quantity* and the *quality* of information available to assess the robustness of structural inferences (i.e., estimates, revenue predictions and optimal reserve price recommendations). We show that the latter are sensitive to the *quantity* of information when *quality* is low such as in field settings, and that improving *quality* in such settings dampens the effect of *quantity* and unveils out-of-equilibrium bidding patterns. Yet, a counterfactual analysis of the seller's revenues and optimal reserve prices indicates that behavior is best explained by the usual Nash equilibrium model with either risk aversion or a power form of probability misperception. When the information available is of the highest *quality*, as in laboratory settings, this model is typically rejected because of a nonlinear bidding behavior. We consider two rationales for such behavior and find that they leave revenue predictions and optimal price recommendations hardly affected.

Keywords: first-price sealed bid auctions, structural econometrics of auctions, constant relative risk aversion, probability misperception, expected revenue predictions, optimal reserve prices, experiments.
JEL Classification Numbers: C9, D44, D47, L1

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1 Introduction

The growing use of auctions to allocate assets has been paralleled in the last two decades by an impressive literature on the structural estimation of auction models (see Paarsch and Hong, 2006, and Athey and Haile, 2007, and Reiss and Wolak, 2007, for reviews). In its essence, the structural approach to games of incomplete information consists in recovering unobserved information, such as private signals, by assuming players to act in some equilibrium. In the context of auctions, this approach is fundamental for predicting revenues, implementing a revenue enhancing reserve price or an alternative selling mechanism, but its relevance hinges upon the assumption that the observed bids are in the presumed equilibrium. In this paper, we use laboratory data and manipulate the *quantity* of information available to the researcher (i.e., information about all bids or only about winning bids) and its *quality* (i.e., information or not about private signals and/or about their common probability distribution) to assess the robustness of structural inferences from first-price auction models.

We use data from auction experiments involving three, four or six bidders, and a methodology inspired from Laffont, Ossard and Vuong (1995) and Donald and Paarsch (1996) for the analysis of symmetric first-price auctions with independent private values. Our choice to study symmetric independent private value auctions keeps the analysis simple as it requires the bidders' private values to be independent draws from the same probability distribution and avoids the complications arising from holding private information in common value settings.[1] Further, as in experiments the distribution of values and the number of participating bidders are known to the researcher, we use a parametric approach rather than a nonparametric one to avoid imposing additional conditions for identification and estimation, see e.g., Guerre, Perrigne and Vuong (2000), and Campo, Guerre, Perrigne and Vuong (2011).[2] Our analysis therefore complements the one of Bajari and Hortaçsu (2005) who question the reasonability of structural estimates from auction models with laboratory data. While their study concentrates on estimating various models, among which the Symmetric Bayes-Nash Equilibrium (SBNE) one, with nonparametric methods and on

[1]See Kagel and Levin (2014) for a review, and Armantier (2002) for a structural analysis using experimental data to decide which of the *common value* or the *independent private values* paradigm explains behavior best.

[2]See also Hendricks and Paarsch (1995) and Reiss and Wolak (2007) for discussions of the relative advantages of parametric and nonparametric structural approaches to field auction data.

comparing the goodness-of-fit of the recovered values, ours deals instead with comparing the estimates, revenue predictions and optimal reserve prices of the SBNE model under information conditions that pertain to field or laboratory settings.

We provide identification conditions to estimate the SBNE model with either homogeneous Constant Relative Risk Averse (CRRA) preferences or a power Probability Weighting Function (PWF) in field and experimental settings when bidders have their private values drawn from a Beta distribution. We show that in both settings, and in the absence of information about bidders' probability perceptions or risk aversion preferences, these two behavioral traits cannot be disentangled if bidders play in the SBNE. When the *quality* of information is low, such as in field settings, they also cannot be disentangled from the case where bidders are risk neutral and have their values drawn from a constrained Beta distribution. As a result, revenue predictions and optimal reserve prices in field settings can be dramatically affected by the researcher's interpretation of the model's estimates if bidders' behavior does not comply with the SBNE model's basic assumption of invariance of estimates to the information available or to the number of bidders.

Our study indicates that the interpretation of the model's estimates affects more the determination of optimal reserve prices than revenue predictions, no matter the number of bidders (i.e., 3,4 or 6). It also shows that increasing the *quality* of information available, i.e., disclosing the bidders' value realizations to the researcher without revealing their exact generating process, leads to *(i)* significantly lower CRRA risk estimates, *(ii)* a rejection of the homogeneity hypothesis and *(iii)* a dampened effect of information *quantity* on structural inferences. The data we consider also allows us to check the assumption of independence across market days when studying single-unit auctions with subsamples of data from multi-unit markets (see e.g., Laffont, Ossard and Vuong, 1995, and Brendstrup and Paarsch, 2006). When the *quantity* of information is low, we find evidence of a behavioral spillover from multi-unit auctions to single-unit ones which translates into a less aggressive bidding. This, however, applies to markets organized as sequential (first-price) auctions, not as discriminatory-price auctions, and its effect vanishes as information *quality* improves.[3] Such patterns are not borne by the SBNE model with homogeneous CRRA preferences or a power PWF and appear to mostly affect the researcher's inferences when the *quality* of informa-

[3]See Bednar, Chen, Liu and Page (2012) for experimental evidence on behavioral spillovers across games.

tion is low. Our analysis shows that in this case, improving information *quality* helps organizing the data and turns this model into a better revenue predictor than the one assuming risk neutrality and no probability misperception.

From an experimental perspective, i.e., when the information available is of the highest *quality*, bidding behavior hardly fulfills the SBNE invariance assumptions and typically rejects the models considered because of a nonlinear behavior which has typically been attributed to bidders' heterogeneous CRRA preferences (see e.g., Cox, Smith and Walker, 1988, Cox and Oaxaca, 1996, Chen and Plot, 1998, and Palfrey and Pevniskaya, 2008).[4] Our approach based on (unconstrained) Beta distributions permits an alternative rationalization in terms of homogeneous probability misperception: risk neutral bidders are misreading the uniform distribution as a Beta one and are immune to probability misperception as defined in Cumulative Prospect Theory. We compare the goodness-of-fits, revenue predictions and optimal reserve price recommendations resulting from this interpretation to those obtained when assuming Prelec's (1998) exponential PWF (which allows for nonlinear SBNE bidding) and find no significant difference to report. As such, either interpretation provides an alternative to the more demanding hypothesis of heterogeneous CRRA preferences to explain the observed nonlinear behavior.

The next section reviews the SBNE model of bidding with independent private values and outlines the econometric procedures used for the estimation frameworks considered. Section 3 describes the data. Section 4 reports the estimation results as well as a counterfactual analysis of revenue predictions and policy recommendations; and Section 5 concludes.

2 Theory and model specifications

2.1 Symmetric Bayes-Nash Equilibrium bidding

Assume n bidders who compete for the purchase of a single commodity to be awarded to the highest bidder for a price equal to her/his bid. Each bidder receives a private value v which is an independent draw from a distribution $F(\cdot)$ with density $f(\cdot)$ defined on $[\underline{v}, \overline{v}]$. Bidders have the same utility $u(\cdot)$ on monetary payments, with

[4] See Battigali and Siniscalchi (2003) for a theoretical explanation of less-than-proportional bidding at high values in terms of "rationalizable bidding".

$u' > 0$, $u'' \leq 0$ and $u(0) = 0$ so that a bidder with value v who submits a bid b has a utility of winning the auction equal to $u(v - b)$. The number of bidders n, the distribution $F(\cdot)$ and the utility $u(\cdot)$ are common knowledge but value realizations are private information. In this context, a bidding strategy $b(\cdot)$ is a SBNE strategy if it is a best response for bidder i to use $b(\cdot)$ when all bidders $j \neq i$ also use $b(\cdot)$. Since $b(\cdot)$ is monotone increasing in values (Maskin and Riley, 2000), $b^{-1}(b_i)$ stands for its inverse and bidder i's expected payoff is defined as

$$U(v_i, b_i) = u(v_i - b_i)F\left(b^{-1}(b_i)\right)^{n-1}. \quad (1)$$

Using the Bayes-Nash best-response first-order condition and imposing a symmetric behavior, $b_i = b(v_i), \forall i = 1, \ldots, n$, we get the following nonlinear first order differential equation

$$b'(v_i) = \frac{\mathrm{d} \ln F(v_i)^{n-1}}{\mathrm{d} \ln u\left(v_i - b(v_i)\right)}, \quad (2)$$

which has for boundary condition $b(p) = p$, with $p \in [\underline{v}, \overline{v}]$ standing for the seller's (common knowledge) reserve price. Following Holt (1980), bidders can be assumed to display homogeneous constant relative risk averse preferences in which case $u(w) = w^{c_r}$, with $c_r > 0$ representing the buyers' common (knowledge) Arrow-Pratt index of constant relative risk aversion. With such preferences, (2) yields the following SBNE bidding strategy

$$b_r(v_i, F, p, c_r) = v_i - \int_p^{v_i} \left[\frac{F(\vartheta)}{F(v_i)}\right]^{\frac{n-1}{c_r}} \mathrm{d}\vartheta. \quad (3)$$

Alternatively bidders can be assumed to commonly misperceive probabilities as in Cumulative Prospect Theory (Kahneman and Tversky, 1979); see Goeree, Holt and Palfrey (2002) and Armantier and Treich (2009a, 2009b) for applications to first-price auction models. Assuming risk neutrality and using a power Probability Weighting Function (PWF) $\Phi(\pi) = \pi^{c_\alpha}$ where $c_\alpha > 0$ stands for the distortion parameter, (1) becomes

$$U(v_i, b_i) = (v_i - b_i)\left[F\left(b^{-1}(b_i)\right)^{n-1}\right]^{c_\alpha},$$

and the SBNE bidding strategy is then

$$b_\alpha(v_i, F, p, c_\alpha) = v_i - \int_p^{v_i} \left[\frac{F(\vartheta)}{F(v_i)}\right]^{c_\alpha(n-1)} \mathrm{d}\vartheta. \quad (4)$$

It follows that for $0 < c_r < 1$ or $c_\alpha > 1$, (3) and (4) are linear in values if F is uniform on $[\underline{v}, \overline{v}]$, and they imply greater bids than those predicted for risk neutral bidders who are immune to probability misperception (see Armantier and Treich, 2009b).

An important variable for policy making is the seller's expected revenue which, in the context of first-price auctions, is determined by the expected winning bid, so we have

$$\mathrm{ER}(F, p, c_\mu) = \int_p^{\overline{v}} b_\mu(\vartheta, F, p, c_\mu) \mathrm{d}F(\vartheta)^n, \tag{5}$$

with $c_\mu = \{c_r, c_\alpha\}$ and p being optimal at p^* if it maximizes (5). Actually, the optimal reserve price p^* is the solution to $0 = p^* - [1 - F(p^*)] / f(p^*)$ if c_μ equals 1, and it must be numerically determined from (5) if not. Two interesting properties of this optimal reserve price are that it is invariant to n if bidders are risk neutral and that it decreases with risk aversion (Riley and Samuelson, 1981) — it is straightforward to show that it also decreases when risk neutral bidders misperceive probabilities with $c_\alpha \in [1, \infty)$. Thus, as far as structural methods are to be used for policy making, it is of utmost importance for the seller to exactly know the buyers' preferences or behavioral traits, for casting the wrong assumption would effectively decrease the seller's expected revenue.

Notice finally that the assumption of symmetric bidders is central to the model's predictions and leads to a unique equilibrium strategy.[5] The diagnosis of a heterogeneous bidding behavior in the above SBNE context has typically been attributed to differences in the bidders' risk preferences or probability misperceptions; and how these differences are modelled matters considerably. On the one hand, we may relax the common knowledge assumption that all bidders have the same parameter c_r or c_α and assume instead that they exactly know each other's different parameters $c_{i\mu}$ with $i = 1, ..., n$ and $\mu = \{r, \alpha\}$. This, however, would lead to an asymmetric model *à la* Li and Riley (2007) which would not suit the analysis of symmetric auctions.[6] On the other hand, bidders may be assumed to have private information about their own parameter and to share a common prior about the distribution from which these parameters are independently drawn, see Van Boening, Rassenti and Smith (1998).

[5] Maskin and Riley (2003) actually show that uniqueness requires that the bidders' common distribution of values has a positive, even infinitesimally small, mass at $v = \underline{v}$ to cope with a singularity at this value.

[6] See Athey and Haile (2007) and Campo (2012) for the semi-parametric identification conditions needed for this type of asymmetry.

While such an *ex-ante* symmetric model is sometimes believed to successfully organize behavior in first-price auction experiments (see e.g., Cox et al., 1988, Cox and Oaxaca, 1996, Chen and Plott, 1998, and Palfrey and Pevniskaya, 2008, and Armantier and Treich, 2009b), it should be noted that the required common knowledge assumption of the distribution of parameters bears heavily on the determination of SBNE strategies since these can only be defined over a range of bids which is itself determined by the distribution and its support, both being unknown to experimental subjects (see Harrison, 1990).[7] As policy recommendations for symmetric auction settings typically assume homogeneous bidders, we focus analysis on the SBNE model for homogeneous bidders presented above.[8]

2.2 Estimation frameworks and scenarios

We consider three estimation frameworks which assume the number of participating bidders n and the support of the distribution F to be known to the researcher. The first framework is most relevant to field studies in that it assumes the researcher to be unaware of the bidders' private values and to have incomplete information about the distribution F. The second assumes the researcher to still have incomplete information about F but to know the bidders' value realizations. Although such knowledge preempts the use of structural methods to recover the unknown distribution F, it allows us to highlight the sensitivity of structural estimates to the information available. The third framework pertains to experimental studies in that the researcher knows both the distribution F and the bidders' value realizations. Henceforth, we will refer to these settings as the Empirical, the Hybrid and the Experimental frameworks, respectively.

[7] In the SBNE of this game, a bidder's expected utility should account for the fact that s/he has the highest of n values as well as being the least risk averse of the n bidders. Van Boening et al. (1998) show that for at least some range of values, a high-valued bidder with low risk aversion cannot be disentangled from a low-valued one who is highly risk averse so that SBNE bidding strategies cannot be determined over this range. This, in turn, prevents identification, see Athey and Haile (2007). Chen and Plott (1998) overcome this indeterminacy problem by assuming bidders to compare their individual risk parameter to the *expected* risk parameter. Armantier and Treich (2009b) use a similar argument for their QRE analysis of discrete bids which assumes heterogeneity in both risk parameters and two probability misperception parameters; the three parameters being drawn from a trivariate distribution which is assumed to be common knowledge.

[8] Note that the study of 'unobserved heterogeneity' in empirical auction investigations assumes bidders to rationally use some information which the researcher has no access to (see e.g., Hong and Paarsch, 2006, p.156). To the extent that participants in auction experiments have no such information, this approach has no bite for the analysis of laboratory data.

Each framework implies its own econometric model specification and assumes the researcher to consider estimating models of bids b_{it} with a sample of T observations. These models assume F to belong to the Beta family of distributions which nests a wide variety of distributions, including the uniform and asymmetric ones, defined by the parameter vector $\theta = (\theta_1, \theta_2)$.[9] Although we cast a parametric structural approach like Donald and Paarsch (1996), we estimate our models with an additive error term which entails the use of Nonlinear Least Squares methods, like Laffont et al. (1995). As this approach allows us to directly assess the estimates' invariance to the number of bidders and how its possible violation affects structural inferences, we find it to suit our goal best. Before considering the specifics of each estimation framework, we note that we will refer to the Beta distribution F by its parameter vector θ and will mostly deal with the case of no binding reserve price (i.e., $p = 0$) so the bidding functions (3) and (4) will be denoted by $b_\mu(v_i, \theta, c_\mu)$ with $\mu = \{r, \alpha\}$, and their generic estimating equation will be defined as

$$b_{it} = b_\mu(v_{it}, \theta, c_\mu) + \varepsilon_{it} \qquad \text{with } i = 1, ..., n,\ t = 1, ..., T \\ \text{and } E(\varepsilon_{it}|v_{it}) = 0. \tag{6}$$

For each estimation framework we consider the following two scenarios: one in which the researcher has information pertaining to winning bids (henceforth, the WINBIDS scenario) and the other in which s/he has information pertaining to all submitted bids (the ALLBIDS scenario). Clearly, if individuals bid in the SBNE, then the distributions' estimates (θ_1, θ_2) of both scenarios are consistent in the sense that they converge to the same value as the sample of observations increases. However, with data samples of finite size we expect differences to be observed simply because the winning bid is only the maximum statistic of the sample of bids $B = \{b_1, b_2, \ldots, b_n\}$ and contains less information about the parameter vector $(\theta_1, \theta_2, c_\mu)$ than B. The estimates are therefore expected to be similar across scenarios and less volatile in the ALLBIDS scenario. Hence, by testing the equality of WINBIDS and ALLBIDS estimates, we can assess the first-moment effect of the *quantity* of information on structural inferences. Furthermore, since the SBNE bidding strategy is monotone increasing in values, the sample B contains as much information as the vector of

[9]The functional form of the Beta density is $f(v) = \frac{\Gamma(\theta_1)\Gamma(\theta_2)}{\Gamma(\theta_1+\theta_2)} v^{\theta_1 - 1}(1-v)^{\theta_2 - 1}$ for $v \in (0,1)$ and with $\theta_1, \theta_2 > 0$.

value realizations for the estimation of $(\theta_1, \theta_2, c_\mu)$ so we also conjecture that differences in the estimates across frameworks must asymptotically disappear if bidders play in the SBNE.[10] In sum, if bidders' play in the SBNE, then the effect of the *quality* of information, like that of its *quantity*, should leave structural inferences unchanged.

2.3 Inference from the Empirical framework

As in this framework the researcher has only access to bid data, (6) cannot be estimated and one has to consider instead the bids' conditional first-order moments

$$\begin{aligned} & E\left(b_t^W|\theta, c_\mu\right) = \textstyle\int_0^1 b_\mu\left(\vartheta, \theta, c_\mu\right) \mathrm{d}F_\theta(\vartheta)^n, \\ \text{and} \quad & \\ & E\left(b_{it}|\theta, c_\mu\right) = \textstyle\int_0^1 b_\mu\left(\vartheta, \theta, c_\mu\right) \mathrm{d}F_\theta(\vartheta). \end{aligned} \tag{7}$$

These first-order moments can be used to propose the following nonlinear regression models for the WINBIDS and the ALLBIDS scenarios

$$\begin{aligned} & b_t^W = E\left(b_t^W|\theta, c_\mu\right) + \varepsilon_t^W \\ \text{and} \quad & \\ & b_{it} = E\left(b_{it}|\theta, c_\mu\right) + \varepsilon_{it}^A \end{aligned} \tag{8}$$

with $i = 1, ..., n$ and $E(\varepsilon_t^W) = E(\varepsilon_t^A) = 0$. The following proposition gives the identification condition needed to estimate these models.

Proposition 1. *The models in (8) are not first-order identified. Partial identification can be achieved under the restrictions* $c_\mu = K > 0$ *and* $\theta_2 = 1$*. With such restrictions, an estimate of* c_r *(*c_α*) obtains from* $K/\hat{\theta}_1$ *(*$\hat{\theta}_1$*).*

Proof: See Appendix A.

As a result of Proposition 1, we get a simple linear model that allows us to test for SBNE bidding with either homogeneous CRRA preferences or a power PWF. By setting (without loss of generality) the restriction $c_\mu = K = 1$, we have $b_r(v, (\theta_1, 1), 1) =$

[10]This is because if a statistic S is sufficient for a parameter vector $(\theta_1, \theta_2, c_\mu)$ and h is a bijective function, then $h(S)$ is also sufficient (see Gouriéroux and Monfort, 1995). In our case the inverse bidding function $v_i = b^{-1}(b_i)$ is bijective and meets this condition.

$b_\alpha(v,(\theta_1,1),1) = \frac{n-1}{n-1+1/\theta_1}v$, so (8) reads

$$b_t^W = \frac{n-1}{n-1+1/\theta_1} E_{v_{(1)}}(v) + \varepsilon_t^W \tag{9}$$

and

$$b_{it} = \frac{n-1}{n-1+1/\theta_1} E(v) + \varepsilon_{it}^A \tag{10}$$

where the expectation in the WINBIDS case, $E_{v_{(1)}}(v)$, is taken with respect to the distribution of the first-order statistic of n draws from $F_{(\theta_1,1)} = B(\theta_1, 1)$.

2.4 Inference from the Hybrid and Experimental frameworks

As in both frameworks, the value realizations are known to the researcher, (6) can *a priori* be estimated. Note that the Hybrid framework relates to the Empirical one in that F is partially known and must still be recovered from the data whereas the knowledge of F in the Experimental framework precludes such exercise and allows a direct test of the SBNE model. Our approach here consists in encompassing (6) into a general model that involves two parameters, $\theta = (\theta_1, \theta_2)$, and that allows for nonlinear bidding if $\theta_2 \neq 1$. We therefore consider estimating the following equations for the WINBIDS and ALLBIDS cases.

$$\begin{aligned} & b_t^W = b_\mu(v_{(1)t}, \theta, c_\mu) + \varepsilon_t^W \\ \text{and} \quad & \\ & b_{it} = b_\mu(v_{it}, \theta, c_\mu) + \varepsilon_{it}^A \end{aligned} \tag{11}$$

The following proposition shows that by increasing the *quality* of the information available when private values are observed, identification only requires c_μ to be constrained.

Proposition 2. *The nonlinear models in (11) are not first-order identified in the Hybrid and Experimental frameworks. Partial identification can be achieved under the restriction $c_\mu = K > 0$. In the Experimental framework, if $\hat{\theta}_2 = 1$ then the SBNE model is not rejected and an estimate of c_r (c_α) obtains from $K/\hat{\theta}_1$ ($\hat{\theta}_1$).*

Proof: See Appendix A.

By setting (without loss of generality) the restriction $c_\mu = K = 1$ in (11), we thus get

$$b_t^W = v_{(1)t} - \int_0^{v_{(1)t}} \left[\frac{F_\theta(\vartheta)}{F_\theta(v_{(1)t})} \right]^{n-1} \mathrm{d}\vartheta + \varepsilon_t^W, \tag{12}$$

and

$$b_{it} = v_{it} - \int_0^{v_{it}} \left[\frac{F_\theta(\vartheta)}{F_\theta(v_{it})} \right]^{n-1} \mathrm{d}\vartheta + \varepsilon_{it}^A, \tag{13}$$

which can be estimated with Nonlinear Least Squares methods using quadrature numerical procedures for integration and the outer product of gradients for estimating the covariance matrix.

To highlight the effect of improving the *quality* of information alone on inferences from the Empirical framework, we impose the restriction $\theta_2 = 1$ when assuming the Hybrid framework but not when assuming the Experimental framework. In the latter case, if we cannot reject the null $\theta_2 = 1$, then the estimated bidding strategy is linear in valuations with a slope equal to $(n-1)/(n-1+c_r)$ or $(n-1)/(n-1+1/c_\alpha)$, as predicted by the SBNE model with uniformly drawn values. Nonlinear bidding is characterized by $\theta_2 \neq 1$ and interpreted as evidence against the SBNE model with uniform values. We can thus assess the SBNE model for either CRRA preferences or a power PWF in these frameworks by testing

$$H_0: \begin{bmatrix} \theta_1^0 \\ \theta_2^0 \end{bmatrix} = \begin{bmatrix} c_\mu^0 \\ 1 \end{bmatrix} \text{ vs. } H_1: \begin{bmatrix} \theta_1^0 \\ \theta_2^0 \end{bmatrix} \neq \begin{bmatrix} c_\mu^0 \\ 1 \end{bmatrix}, \tag{14}$$

with $c_\mu^0 = \{1/c_r^0, c_\alpha^0\}$. The null hypotheses in (14) define functions h_μ such that if the model holds and c_r^0 or c_α^0 is the true constant relative risk aversion or probability distortion parameter, then there is an implicit restriction on the parameter space of vector θ which is given by $\theta^0 = h_\mu(c_\mu^0)$. In Appendix A, we show that to test (14), one only needs to conduct a standard normal test of the significance of the second log-parameter of the estimated Beta distribution.

3 Data

We estimate our models with the data of Isaac and Walker (1985, henceforth IW), Cox, Smith and Walker (1988, henceforth CSW), Dyer, Kagel and Levin (1989, henceforth DKL) and Hanaki, Neugebauer and Pezanis-Christou (2016, henceforth HNPC) which we briefly review.

The IW data consist of ten independent sessions of auctions with four bidders who played for 25 rounds. These data have been used by several studies on overbidding in first-price auction experiments (e.g., Cox, Smith and Walker, 1988, Cox and Oaxaca, 1996, Ockenfels and Selten, 2001, and Palfrey and Pevniskaya, 2008).

The DKL data consist of three independent sessions in which bidders played in auctions with either three or six bidders. Participants submitted contingent bids in each round of play, namely one for a market with three bidders and another for a market with six bidders. The prevailing market size for a particular round was then randomly drawn with equal chance, and the corresponding bid validated. Such a within-subject protocol generates a variation in the number of bidders that allows the nonparametric identification of the SBNE model for CRRA bidders, which Bajari and Hortaçsu (2005) estimated with the last 23 rounds of each session. We analyze the markets for three and six bidders separately as well as in the aggregate to assess the estimates' invariance condition needed for the nonparametric identification of first-price auction models.

The CSW data consist of three independent sessions of auctions with three bidders who played for 20 rounds. These data involve experienced bidders who already played in previous first-price auction experiments; we use them to complement the DKL data on markets with three bidders.

HNPC study multi-unit high-bid auctions with six bidders and with the number of units sold in each round varying from one to three. Participants in these experiments had unit-demands and played for 100 rounds in each of which the number of units for sale could be one (with probability 1/2), two (with probability 1/4) or three (with probability 1/4). As the exact supply available for sale was revealed at the outset of each round, these experiments reproduce the feature of supply variability that characterizes most real-world auctions that are held on a frequent basis, such as auctions for perishables (see e.g., Laffont et al., 1995, Pezanis-Christou, 2001, Van der Berg and Van Ours, 2001, and Brendstrup and Paarsch, 2006). We thus consider subsamples of rounds in which only one unit is sold (as in Laffont et al., 1995, and Brendstrup and Paarsch, 2006) to evaluate the assumption of independence across market days when estimating auction models with such subsamples. Since sequential first-price and discriminatory auctions are strategically identical when there is only one unit for sale, the study of these subsamples will document the impact of market organization on behavior in single-unit first-price auctions. The data we consider consist of 2×6 independent sessions with a number of rounds varying between 41 and 57 but we focus analysis on the first 10 and last 25 rounds to account for bidders' experience and for consistency with the length of the other series studied.

The protocol details of the above experiments are summarized in Table 1. All bids and value realizations have been rectangularized to the unit interval to fit the support of the Beta distribution F. For expository convenience, we often refer to these datasets by the number of bidders involved so $n = 4$ refers to IW, $n = 3^*$or 6^* to DKL, $n = 3x$ to CSW, and $n = 6d$ or $6s$ to HNPC, with d for discriminatory and s for sequential.

Table 1: Details of Experimental Designs.

Dataset	n	S^a	$[\underline{v}, \overline{v}]$	Matching	Dual Market	Info Feedback	Rounds/ Session	T^b	Exch. Rate
DKL	3|6	3	[\$0, \$30]	Random	Yes	Full[c]	23	414	\$1/\$
CSW	3^d	3	[\$0, \$6]	Partner	No	Win.bid	20	180	\$1/\$
IW	4	10	[\$0, \$10]	Partner	No	Win.bid	25	1000	\$1/\$
HNPC(d)	6	6	$[E0, E100]$	Partner	No	Win.bid	(41,57)[e]	1704	€0.05/E
HNPC(s)	6	6	$[E0, E100]$	Partner	No	Win.bid	(42,56)	1770	€0.05/E

Note: [a]: Number of independent sessions; [b]: Total number of observations; [c]: All bids and values; [d]: Experienced Bidders ; [e]: (Min, Max)

4 Results

We analyze each framework separately and check whether the data fulfill the SBNE model's basic behavioral assumptions and implications outlined in sections 2.1 and 2.2.

We first check for a homogeneous behavior by including individual dummy variables for the parameter θ_1 in the Empirical and Hybrid frameworks and for both θ_1 and θ_2 in the Experimental framework. With θ_{1i} standing for bidder i's θ_1-parameter, we define dummies δ_{1i} that take the value of 1 if the bid is submitted by bidder i and 0 otherwise and we specify $\theta_{1i} = \theta_1 + \sum_{j=2}^{nS} \tau_{1j}\delta_{1j}$ for $1 < i \leq nS$ where nS stands for the number of bidders in a session times the number of independent sessions (c.f., Table 1). Homogeneous bidding in the Empirical or Hybrid framework then implies $\tau_{1i} = 0$ for all $1 < i \leq nS$. We proceed in a similar way for the Experimental framework by defining in addition $\theta_{2i} = \theta_2 + \sum_{j=2}^{nS} \tau_{2i}\delta_{2j}$ for $1 < i \leq nS$ and testing the joint hypothesis $\tau_{1i} = \tau_{2i} = 0$. This procedure is used for both the WINBIDS and the ALLBIDS scenarios and we shall refer to the null of this test as H_0^H.

To assess the effect of the *quantity* of information on the estimates, we compare the θ_1-estimates of the WINBIDS and ALLBIDS scenarios in the Empirical and Hybrid frameworks using a z-test, and in the Experimental framework using a $\chi^2(2)$-test. We refer to the null of this test as $H_0^{\approx}$, and note that since the estimates are not independent across scenarios, the tests' critical values only represent a lower bound for the true values.[11] Next, to assess the estimates' equality across *(i)* frameworks (i.e., the effect of information *quality*), *(ii)* the number of bidders in the DKL data (c.f., the invariance of risk estimates) and *(iii)* market organizations in the HNPC data (c.f., the presence of a behavioral spillover) we check if their 95% confidence intervals overlap or not.

Finally, as in our setting the field researcher can assume private values to be drawn either from $B(\hat{\theta}_1, 1)$ (in which case $c_r = 1$ or $c_\alpha = 1$) or from $B(1, 1)$ (in which case $c_r = 1/\hat{\theta}_1$ or $c_\alpha = \hat{\theta}_1$). We will refer to these alternatives as the BETA and the UNI condition, respectively, and we conduct a counterfactual analysis of expected revenues and optimal reserve prices to figure out which is most relevant for policy making. Using (5), the seller's expected revenues for the Empirical and the Hybrid frameworks under each condition are thus defined as

$$\mathrm{ER}_B((\hat{\theta}_1, 1), p, 1) = \int_p^1 b_\mu(\vartheta, (\hat{\theta}_1, 1), p, 1)\mathrm{d}F_{(\hat{\theta}_1,1)}(\vartheta)^n \tag{15}$$

and

$$\mathrm{ER}_U((\hat{\theta}_1, 1), p, c_\mu) = \int_p^1 b_\mu(\vartheta, (\hat{\theta}_1, 1), p, c_\mu)\mathrm{d}F_{(1,1)}(\vartheta)^n \tag{16}$$

where $b_\mu(\cdot)$ with $\mu = \{r, \alpha\}$ is defined by (3) or (4). And since in the Experimental

[11] Unfortunately we cannot build a Haussman-type of test for this necessary condition because the estimates would be biased both in ALLBIDS and in WINBIDS, neither can we test the smaller variance in ALLBIDS because the variances' asymptotic distributions are of unknown forms.

framework the researcher knows that $F(v) = B(1,1)$, revenue predictions can only be determined under the UNI condition by (16) provided that the null $\hat{\theta}_2 = 1$ is not rejected when estimating (11), c.f. Proposition 2. In case it is rejected, we consider a Hindsight prediction that assumes $F(v) = B(1,1)$ and the nonlinear bidding strategy $b_\mu(s, (\hat{\theta}_1, \hat{\theta}_2), p, 1)$. This prediction reverts to assuming that bidders are immune to probability misperception as defined in Cumulative Prospect Theory and to bid in the SBNE while misperceiving the uniform distribution as a Beta one with parameters $(\hat{\theta}_1, \hat{\theta}_2)$. In this case, expected revenues are defined as

$$\mathrm{ER}_H((\hat{\theta}_1, \hat{\theta}_2), p, 1) = \int_p^1 b_\mu(\vartheta, (\hat{\theta}_1, \hat{\theta}_2), p, 1) \mathrm{d}F_{(1,1)}(\vartheta)^n \tag{17}$$

4.1 Empirical framework

Table 2 reports the estimation outcomes of (9) and (10). The figures indicate no rejection of the null of homogeneity H_0^H at $\alpha = 0.05$. The $\hat{\theta}_1$-estimates are all significantly greater than 1 and would thus accommodate SBNE bidding with either *(i)* uniformly drawn values and homogeneous CRRA preferences or a power form of probability misperception or *(ii)* values drawn from a $B(\hat{\theta}_1, 1)$ distribution, risk neutral preferences and immunity to probability misperception. Both interpretations generate the same goodness-of-fit which we measure in terms of the Akaike Information Criterium (AIC) and find it to be best under the WINBIDS scenario for all n. The estimated distributions of values and the corresponding bidding functions are displayed in Appendix B.

Evidence of out-of-equilibrium behavior comes from the rejection of the null of equal $\hat{\theta}_1$-estimates across scenarios, $H_0^{\approx}$, for all n except $n = 3^*$ and 6^* so the *quantity* of information available usually matters. A comparison of the estimates' confidence intervals for $n = 3^*$ to those for $n = 6^*$ reveals a significant difference in the ALLBIDS scenario; the risk parameter estimates being equal to $c_r = .709$ for $n = 3^*$ and .855 for $n = 6^*$. Since these auctions were attended by the same bidders, such estimates do not support the invariance condition used for the nonparametric identification of the SBNE model. They are also not in line with the nonparametric aggregate risk parameter of $c_r = .158$ of Bajari and Hortaçsu (2005) who used the same data, neither with their estimate of $c_r = .224$ (.247) when the upper and lower 5% (25%) of the data are discarded.[12] When pooling the data for $n = 3^*$ and 6^* we get an aggregate $\hat{\theta}_1$-estimate of 1.29, or equivalently $c_r = .775$, c.f., Appendix C. Such a three- to five-fold difference in the c_r estimates reveals a sensitivity of structural inferences to the econometric procedures used and suggests that the estimation of the distribution's support that characterizes nonparametric approaches leads to more volatile estimates than alternative parametric methods when behavior is out-of-equilibrium.

As for the possible effect of market organization on behavior, we find a significant difference in the WINBIDS estimates for $n = 6d$ and $6s$. Since these two treatments are identical when there is only one unit for sale, such different estimates could witness a behavioral spillover from bidders' participation in rounds with multiple units. If so, then we expect no significant difference between these treatments at the outset of a session. The estimates for the first 10 single-unit rounds reported in the lower panel of Table 2 support this conjecture and suggest that the spillover is most salient in markets organized as sequences of auctions: bidders' participation in sequential auctions leads to lower winning bids in single-unit auctions whereas their participation in discriminatory auctions leaves them unchanged.

Table 2: Estimation outcomes – Empirical framework.

		WINBIDS			ALLBIDS			
Data	$[n]$	$H_0^{H\,a}$	θ_{1W} [95% c.i.]	AIC^b $[T]$	H_0^H	θ_{1W} [95% c.i.]	AIC^b $[T]$	$H_0^{\approx c}$
DKL	[3*]	n.a.	1.53 [1.24, 1.82]	-3.30 [54]	.643	1.41 [1.30, 1.52]	-2.85 [414]	.446
CSW	[3x]	.276	1.55 [1.33, 1.77]	-3.83 [60]	.564	1.26 [1.11, 1.41]	-2.82 [180]	.029
IW	[4]	.072	1.43 [1.31, 1.54]	-3.78 [250]	.960	1.19 [1.12, 1.25]	.2.74 [1000]	.000
DKL	[6*]	.187	1.22 [1.08, 1.37]	-4.62 [69]	.894	1.17 [1.07, 1.27]	-2.76 [414]	.552
HNPC	[6d]	n.a.	1.39 [1.24, 1.54]	-4.14 [150]	.431	1.18 [1.10, 1.25]	-2.60 [900]	.011
HNPC	[6s]	.208	1.15 [1.06, 1.24]	-4.53 [150]	.427	1.08 [1.02, 1.15]	-2.74 [900]	.024
HNPC*	[6d]	n.a.	1.38 [1.16,1.60]	-4.31 [60]	.994	1.18 [1.07,1.30]	-2.65 [360]	.011
HNPC*	[6s]	n.a.	1.35 [1.16,1.53]	-4.58 [60]	.001	1.02 [.92,1.12]	-2.65 [360]	.001

Note: ^{a}: Rejection probability of the null of homogeneity of the $(n \times S)$ $\tilde{\theta}_1$ estimates; ^{b}: Akaike Information Criterium, $AIC = 2/T + ln(SSR/T)$ with T standing for the number of observations; ^{c}: Rejection probabilities of the null $\theta_{1W} = \theta_{1A}$ according to z-test.

Table 3 reports the observed and predicted revenues defined by (15) and (16) (assuming $p = 0$ and the estimates of Table 2) with their 95% confidence intervals – see

[12] Bajari and Hortaçsu (2005) also estimate a QRE model that assumes a uniform or a Beta distribution of values and which yields risk estimates of .27 and .25, respectively. However, their QRE model is not identified so these estimates should be taken with caution.

Appendix D for a graphical display. As revenues are determined by winning bids, we focus on the WINBIDS predictions and report on the ALLBIDS ones to check for the effect of improving the *quantity* of information on predictions. Expected revenues are perfectly predicted in the WINBIDS condition when assuming the BETA condition, and they are systematically and significantly under-estimated when assuming the UNI condition. As such, the constrained Beta distribution organizes behavior better than the combined assumptions of a uniform distribution and either CRRA preferences or a power PWF that characterize the UNI condition. Hence, our analysis suggests that when the exact functional form of F is unknown to the researcher, assuming risk neutrality (and a constrained Beta distribution of values) may well be a better option for the seller than assuming CRRA preferences or probability misperception (and a uniform distribution of values). This, however, should be taken with caution for two reasons. First, the 'better' BETA predictions come with larger confidence intervals that often overlap those of the UNI predictions, thereby suggesting no significant difference. Second, assuming a particular condition (UNI or BETA) and improving the *quantity* of information typically worsens revenue predictions; a pattern that confirms the presence of an out-of-equilibrium behavior.

Using the WINBIDS estimates of Table 2, Figure 1 displays the expected revenues (15) and (16) as functions of the seller's reserve price p, with their respective maxima characterizing the optimal reserve price p^*. As expected, the latter tend to be greater than those predicted in the SBNE with $c_\mu = 1$ (because the $\hat{\theta}_1$-estimates imply a rightward-shifted distribution of values), and smaller when $0 < c_r < 1$ (as predicted by Riley and Samuelson, 1981) or $c_\alpha > 1$.

In sum, we find weak support for the SBNE model of bidding when assuming a field setting in which the researcher has limited information about the bidders' distribution of values: *(i)* the *quantity* of information available usually matters, *(ii)* bidders' estimated risk parameters are not invariant to the number of bidders, *(iii)* parametric and nonparametric estimation procedures yield very different risk parameter estimates, and *(iv)* there is a behavioral spillover from sequential (multi-unit) to single-unit auctions. Yet, the observed behavior does not reject the null of homogeneity and remains assimilable to one of SBNE bidders with either (1) uniformly drawn values and CRRA preferences or a power form of probability misperception or (2) risk neutral preferences and values distributed according to a Beta distribution. These in-

Table 3: Revenue Predictions – Empirical Framework.

Data	$[n]$	SBNE $c_\mu = 1$	Obs.	WINBIDS UNI [95% c.i.][a]	WINBIDS BETA [95% c.i.]	ALLBIDS UNI [95% c.i.]	ALLBIDS BETA [95% c.i.]
DKL	[3*]	.500	.619	.565	.619	.554	.597
				[.539, .592]	[.569, .669]	[.542, .565]	[.576, .619]
CSW	[3x]	.500	.622	.567	.622	.537	.566
				[.548 ,.586]	[.586 ,.659]	[.518 ,.556]	[.532 ,.600]
IW	[4]	.600	.690	.649	.690	.625	.645
				[.639 ,.658]	[.671 ,.709]	[.617 ,.632]	[.631 ,.659]
DKL	[6*]	.714	.756	.737	.756	.732	.747
				[.724 ,.749]	[.733 ,.779]	[.723 ,.741]	[.730 ,.765]
HNPC	[6d]	.714	.781	.749	.781	.733	.749
				[.739 ,.760]	[.760 ,.801]	[.726 ,.740]	[.736 ,.762]
HNPC	[6s]	.714	.744	.730	.744	.724	.732
				[.721 ,.739]	[.728 ,.760]	[.717 ,.730]	[.719 ,.744]
HNPC*	[6d]	.714	.799	.749	.779	.733	.750
				[.734 ,.764]	[.750 ,.808]	[.723 ,.744]	[.730 ,.770]
HNPC*	[6s]	.714	.775	.746	.775	.717	.719
				[.733 ,.759]	[.750 ,.800]	[.706 ,.729]	[.698 ,.741]

Note: [a]: Confidence intervals computed using the Delta method.

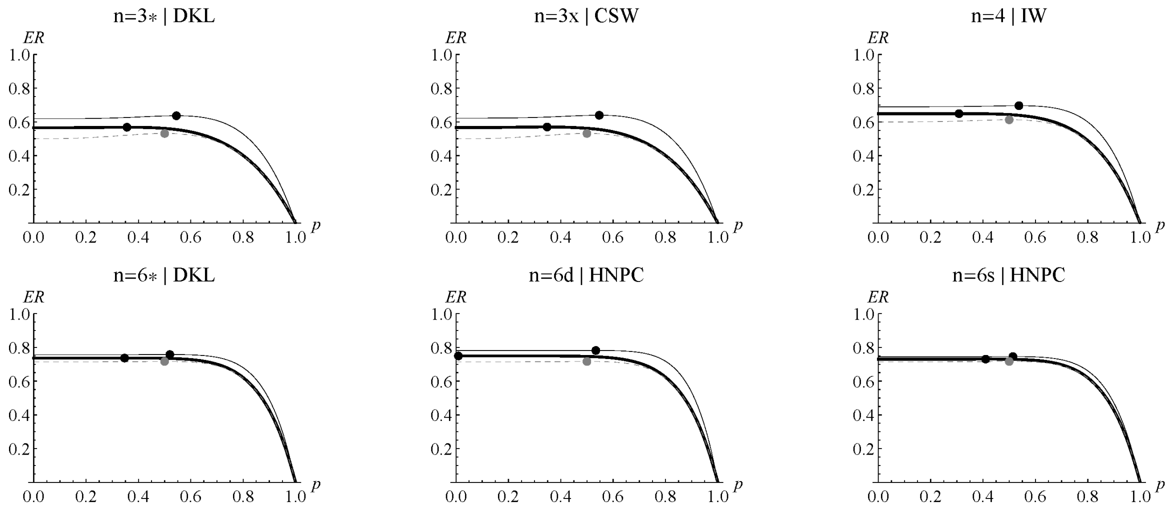

Figure 1: Expected Revenue vs. Reserve Price: Empirical Framework
Bold: UNI, Thin: BETA, Dashed: SBNE (c_μ=1).

terpretations explain bid data equally well and yield similar revenue predictions and optimal reserve price recommendations.

4.2 Hybrid framework

In this framework the researcher knows the bidders' values but still has incomplete knowledge of F so we estimate (12) and (13) with the constraint $\theta_2 = 1$. The outcomes in Table 4 indicate that the better information *quality* of this framework leads to a rejection of homogeneity in almost all cases. The estimates are all significantly greater than 1 and typically larger than those of the Empirical framework which indicates that the *quality* of information matters too. As the null $H_0^{\approx}$ is not rejected for all n at conventional levels of significance and the improved AIC statistics display no clear pattern across scenarios, the improved *quality* also appears to override the effect of the *quantity* of information on inferences.

Table 4: ESTIMATION OUTCOMES – HYBRID FRAMEWORK.

Data	$[n]$	WINBIDS $H_0^{H^a}$	WINBIDS θ_{1W} [95% c.i.]	WINBIDS AIC^b $[T]$	ALLBIDS $H_0^{H^a}$	ALLBIDS θ_{1W} [95% c.i.]	ALLBIDS AIC^b $[T]$	$H_0^{\approx c}$
DKL	[3*]	.007	2.47	-6.28	.000	2.28	-6.27	.181†
			[2.21, 2.72]	[54]		[2.17, 2.39]	[414]	
CSW	[3x]	.001	3.16	-5.77	.000	3.06	-6.24	.746
			[2.65, 3.67]	[60]		[2.76, 3.36]	[180]	
IW	[4]	.000	2.80	-5.40	.000	2.54	-5.26	.161
			[2.49, 3.11]	[250]		[2.34, 2.74]	[1000]	
DKL	[6*]	.057	1.33	-6.39	.000	1.36	-6.78	.640
			[1.20, 1.45]	[69]		[1.29, 1.43]	[414]	
HNPC	[6d]	n.a.	2.59	-6.11	.000	2.78	-6.20	.372
			[2.24, 2.93]	[150]		[2.55, 3.01]	[900]	
HNPC	[6s]	.000	1.72	-6.52	.000	1.65	-6.17	.385
			[1.58, 1.85]	[150]		[1.56, 1.74]	[900]	
HNPC*	[6d]	n.a.	2.51	-6.15	.000	1.93	-5.78	.024
			[2.01, 3.01]	[60]		[1.71, 2.15]	[360]	
HNPC*	[6s]	n.a.	1.74	-5.35	.000	1.14	-4.82	.001
			[1.36, 2.11]	[150]		[.99, 1.28]	[360]	

Note: ^{a}: Rejection probability of the null of homogeneity of the $(n \times S)$ $\tilde{\theta}_1$ estimates; ^{b}: Akaike Information Criterium, $AIC = 2/T + ln(SSR/T)$ with T standing for the number of observations; ^{c}: Rejection probabilities of the null $\theta_{1W} = \theta_{1A}$ according to z-test.

The conduct of Anderson-Darling tests overwhelmingly rejects the null that bidders' actual distributions of values are stochastically equivalent to the estimated $B(\hat{\theta}_1, 1)$ ones for all n (p-values $< .001$), thereby implicitly supporting the UNI scenario. When compared to previous studies, our ALLBIDS risk estimate of .394 for $n = 4$ is in range with those of Pezanis-Christou and Romeu (2007) and Palfrey and Pevniskaya (2008) who use the same data and assume heterogeneous CRRA preferences, i.e., .323 (the average estimate of ten sessions) and .297 (the estimate of pooled data), respectively.[13] Otherwise, the bid patterns reported for the DKL and HNPC data in the Empirical framework remain. The DKL estimates for $n = 3^*$ and 6^* are significantly different in both scenarios ($c_r = .405$ and .752 in WINBIDS, .439 and .735 in ALLBIDS, respectively) and yield an aggregate risk parameter of .498 when pooling data (c.f., Appendix C) which is still twice or thrice those of Bajari and Hortaçsu (2005). As for the HNPC estimates, those for $n = 6s$ are significantly

[13]Note that both studies had to trim high-end values (about 10% and 25% of the data, respectively) to comply with the heterogeneous CRRA model for uniformly drawn risk parameters on $[0, 1]$. To this extent, their c_r estimates are only approximates as they pertain to the linear part of bidders' heterogeneous bidding functions.

smaller than those for $n = 6d$ in both scenarios. And since the WINBIDS ones are not significantly different at the outset of a session (as in Table 2), they further support the existence of a behavioral spillover when information *quantity* is low, as in descending-price auctions.

Table 5: Revenue Predictions – Hybrid Framework.

Data	$[n]$	SBNE $c_\mu = 1$	Obs.	WINBIDS UNI [95% c.i.]	WINBIDS BETA [95% c.i.]	ALLBIDS UNI [95% c.i.]	ALLBIDS BETA [95% c.i.]
DKL	$[3^*]$	.500	.619	.624 [.613, .634]	.732 [.710, .754]	.615 [.610, .620]	.716 [.705, .726]
CSW	$[3x]$	.500	.622	.648 [.633, .662]	.781 [.752, .810]	.645 [.636, .654]	.775 [.757, .793]
IW	$[4]$	.600	.690	.715 [.706, .723]	.820 [.803, .838]	.707 [.700, .714]	.805 [.792, .818]
DKL	$[6^*]$	.714	.756	.745 [.737, .752]	.772 [.757, .787]	.747 [.742, .752]	.776 [.767, .786]
HNPC	$[6d]$	.714	.781	.796 [.788, .803]	.872 [.857, .887]	.800 [.795, .804]	.880 [.871, .889]
HNPC	$[6s]$	.714	.744	.768 [.761, .774]	.816 [.804, .829]	.764 [.760, .769]	.810 [.801, .819]
HNPC*	$[6d]$	.714	.779	.794 [.782, .806]	.869 [.845, .892]	.777 [.768, .785]	.834 [.818, .851]
HNPC*	$[6s]$	.714	.775	.769 [.752, .786]	.818 [.785, .852]	.729 [.715, .743]	.742 [.716, .768]

Note: ^{a}: Confidence intervals computed using the Delta method.

As could be foreseen from the $\hat{\theta}_1$-estimates, the expected revenues in Table 5 usually are significantly larger than those of the Empirical framework. Surprisingly, however, the UNI predictions are now more accurate than the BETA ones for all n, although still significantly different from observed revenues (except for $n = 3^*$) and with no significant differences across scenarios. Furthermore, a better information *quality* dramatically affects the researcher's optimal reserve price recommendations as Figure 2 indicates zero reserve prices for the UNI condition and positive ones, greater than in the Empirical framework, for the BETA condition. Since bidders' preferences or traits and the common distribution F are imperfectly known to the field researcher, our analysis therefore suggests that casting the UNI condition helps reducing the risk of implementing a revenue *decreasing* 'optimal' reserve price.

To summarize, improving the *quality* of information in a field setting dampens the effect of information *quantity* on inferences but also yields different estimates which reject the null of homogeneity and reproduce the out-of-equilibrium patterns *(ii)-(iv)* found when assuming an Empirical framework. Despite the evidence of out-of-equilibrium bidding for all n considered, a better information *quality* still makes of the SBNE model with CRRA preferences or a power PWF a better revenue predictor and, perhaps more importantly, a safer option for the recommendation of optimal

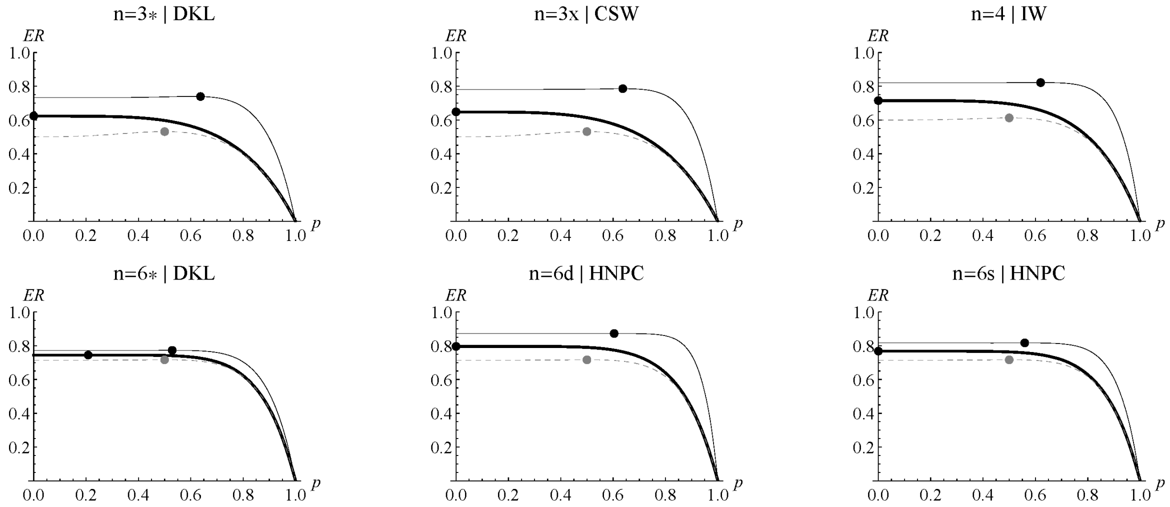

Figure 2: Expected Revenue vs. Reserve Price: Hybrid Framework
Bold: UNI, Thin: BETA, Dashed: SBNE (c_μ=1).

reserve prices than the risk neutral SBNE model.

4.3 Experimental framework

Since in this framework, the researcher knows both F and the value realizations, (12) and (13) are estimated without the constraint $\theta_2 = 1$. When compared to the Hybrid framework, the outcomes in Table 6 suggest that this additional information hardly interferes with the assessment of homogeneity (H_0^H) or with the equality of estimates ($H_0^{\approx}$), nor does it notably improve the model's goodness-of-fit. However, the θ_2-estimates being significantly different from 1 in all cases except $n = 3^*$/WINBIDS and $3x$, it follows from Proposition 2 that the SBNE model is typically rejected because of a nonlinear bidding behavior; see Appendix B for a display of the estimated bidding functions. This holds when estimating aggregate θ_1- and θ_2-estimates for $n = 3$ and 6 with the DKL data, see Appendix C. Such nonlinear (over)bidding behavior has been reported in previous experiments and explained in terms of heterogeneous CRRA preferences. As this assumption requires further *ad hoc* assumptions (c.f. section 2.1) and the trimming of valuable information (the high-end values, c.f., Footnote 13), we consider the following two alternative rationales.

A first 'natural' alternative is to assume bidders to misread the Uniform distribution as a $B(\theta_1, \theta_2)$ with θ_1, $\theta_2 \neq 1$, as we did when defining our Hindsight predictions, c.f. (5). Since this alternative reverts to assuming immunity to probability misperception as defined in Cumulative Prospect Theory, we also consider an exponential PWF proposed by Prelec (1998) defined as $\Phi(\pi) = e^{-\beta(-\ln\pi)^{\alpha}}$with α, $\beta > 0$ and studied by Goeree, Holt and Palfrey (2002) and Armantier and Treich (2009a).[14],[15] While this PWF allows for nonlinear bidding, it is not identifiable in our context and therefore requires the estimation of reduced form models that assume F to be a Uniform distribution on $[0, 1]$. The outcomes are reported in Appendix E and yield similar conclusions regarding H_0^H, $H_0^{\approx}$ and AIC statistics as the unconstrained Beta model so these alternatives appear to organize behavior equally well.

Table 6: Estimation outcomes – Experimental framework.

		WINBIDS				ALLBIDS				
Data	$[n]$	$H_0^{H\,a}$	θ_{1W} [95% c.i.]	θ_{2W} [95% c.i.]	AIC^b $[T]$	$H_0^{H\,a}$	θ_{1W} [95% c.i.]	θ_{2W} [95% c.i.]	AIC^b $[T]$	$H_0^{\approx c}$
DKL	[3*]	.007	3.16 [2.29, 4.04]	1.39 [.95, 1.83]	-6.31 [54]	.000	2.75 [2.47, 3.03]	1.36 [1.17, 1.54]	-6.31 [414]	.304
CSW	[3x]	.001	4.17 [2.26, 6.07]	1.65 [.56, 2.73]	-5.76 [60]	.000	3.59 [2.78, 4.40]	1.41 [.86, 1.96]	-6.24 [180]	.974
IW	[4]	.000	5.15 [3.50, 6.79]	2.13 [1.46, 2.80]	-5.48 [250]	.000	4.44 [3.61, 5.27]	2.28 [1.83, 2.72]	-5.32 [1000]	.022
DKL	[6*]	.004	3.18 [2.08, 4.27]	2.05 [1.53, 2.56]	-6.84 [69]	.000	1.91 [1.70, 2.11]	1.48 [1.33, 1.63]	-6.90 [414]	.199
HNPC	[6d]	n.a.	7.39 [4.10, 10.07]	2.65 [1.72, 3.58]	-6.33 [150]	.000	4.73 [3.92, 5.54]	1.99 [1.68, 2.30]	-6.27 [900]	.681
HNPC	[6s]	.000	3.67 [2.81, 4.52]	2.05 [1.66, 2.44]	-6.84 [150]	.000	2.42 [2.12, 2.72]	1.64 [1.44, 1.84]	-6.22 [900]	.006
HNPC*	[6d]	n.a.	5.22 [2.19, 8.26]	2.03 [1.08, 2.97]	-6.28 [60]	.000	2.70 [2.02, 3.37]	1.53 [1.15, 1.90]	-5.80 [360]	.082
HNPC*	[6s]	n.a.	2.92 [.70, 5.15]	1.64 [.61, 2.66]	-5.36 [60]	.000	1.45 [1.02, 1.88]	1.34 [.94, 1.73]	-4.83 [360]	.029

Note: ^{a}: Rejection probability of the null of homogeneity of the $(n \times S)$ $\tilde{\theta}_1$ and $\tilde{\theta}_2$-estimates; ^{b}: Akaike Information Criterium, $AIC = 4/T + ln(SSR/T)$ with T standing for the number of observations; ^{c}: Rejection probabilities of the null $\theta_{1W} = \theta_{1A}$ and $\theta_{2W} = \theta_{2A}$ according to $\chi^2(2)$ test.

As for the estimates for $n = 3^*$/WINBIDS and $n = 3x$, which do not reject the null $\theta_2 = 1$, i.e., the SBNE model for uniformly drawn values, they yield risk parameters of $c_r = .316$ and of .240 and .279 (WINBIDS and ALLBIDS, respectively). While these are somewhat lower than those of the Hybrid framework ($c_r = .405$, .316 and .327, respectively), they are less than half those of the Empirical framework ($c_r = .654$, .645 and .794, respectively) and thus highlight the sensitivity of structural estimates to information *quality* when behavior mostly complies with the SBNE requirements in a laboratory setting.

[14] In Cumulative Prospect Theory, the transformation $\Phi(\pi)$ applies to the probability of winning $\pi = F(x)^{n-1}$ where $F(x) \approx U[0, 1]$ whereas here, $\Phi(\pi) = \pi \equiv F(x)^{n-1}$ and bidders misperceive $F(x)$ as being $B(\theta_1, \theta_2)$. Clearly, this argument holds only for this framework because the researcher then knows that $F(x)$ is uniform on $[0, 1]$.

[15] See Stott (2006) for a review of the most commonly used PWFs in the Cumulative Prospect Theory literature. Note that since this exponential PWF does not satisfy Armantier and Treich (2009b) 'star-shaped' condition $\Phi(\pi)/\pi \geq \Phi'(\pi)$ for $\pi \in [0, 1]$, it does not imply overbidding for all values $v \in [0, 1]$

Looking at the effect of market organization on behavior, the HNPC estimates for $n = 6d$ and $6s$ in Table 6 indicate no significant difference in the WINBIDS scenario. As this holds for the first 10 rounds, and without significant changes in the estimates, the concern raised by empirical studies of single-unit auctions that assume independence across market meetings (see e.g., Hendricks and Paarsch, 1995, p. 421) seems warranted only when both the *quantity* and the *quality* of information are low. We reach a similar conclusion when comparing the estimates of Prelec's PWF in Table E.1 of Appendix E.

Table 7: Revenue Predictions – Experimental framework.

Data	[n]	SBNE $c_\mu = 1$	Obs.	WINBIDS UNI [a][95% c.i.]	WINBIDS Hindsight [95% c.i.]	WINBIDS Prelec II [95% c.i.]	ALLBIDS UNI [95% c.i.]	ALLBIDS Hindsight [95% c.i.]	ALLBIDS Prelec II [95% c.i.]
DKL	[3*]	.500	.619	.648 [.623, .672]	.628 [.616, .639]	.627 [.602, .653]	n.a.	.614 [.607, .621]	.614 [.602, .626]
CSW	[$3x$]	.500	.622	.670 [.637, .702]	.644 [.628, .659]	.645 [.597, .694]	.658 [.640, .677]	.640 [.629, .650]	.642 [.606, .679]
IW	[4]	.600	.690	n.a.	.713 [.705, .721]	.714 [.696, .731]	n.a.	.693 [.686, .700]	.694 [.681, .706]
DKL	[6*]	.714	.756	n.a.	.750 [.742, .759]	.751 [.737, .764]	n.a.	.738 [.734, .743]	.739 [.734, .744]
HNPC	[$6d$]	.714	.781	n.a.	.796 [.790, .802]	.797 [.785, .809]	n.a.	.788 [.782, .795]	.790 [.780, .800]
HNPC	[$6s$]	.714	.744	n.a.	.764 [.758, .771]	.765 [.758, .771]	n.a.	.750 [.748, .752]	.752 [.746, .758]
HNPC*	[$6d$]	.714	.779	n.a.	.794 [.784, .804]	.794 [.774, .813]	n.a.	.768 [.760, .776]	.769 [.753, .785]
HNPC*	[$6s$]	.714	.775	.802 [.763, .841]	.768 [.750, .785]	.769 [.747, .790]	.753 [.726, .780]	.721 [.703, .739]	.722 [.709, .735]

Note: [a]: Confidence interals computed using the Delta method.

Finally, the revenue predictions in Table 7 indicate that the seldom cases that do not reject the SBNE model (c.f. UNI predictions) yield worse predictions than the Hybrid framework and its lower *quality* of information. We find no significant difference between the Hindsight and the Prelec predictions, no matter the scenario or if the null $\theta_2 = 1$ is rejected. These predictions are also not significantly better than the UNI ones of the Hybrid framework nor do they affect the researcher's optimal reserve price recommendations.

All in all, the SBNE model is typically rejected when assuming a laboratory setting because of a nonlinear overbidding behavior. Such behavior is consistent with the SBNE model for risk neutral bidders if the latter are assumed to misread the uniform distribution as a Beta (non-uniform) one or to display Prelec's exponential form of probability misperception. These interpretations, however, do not outperform the homogeneous CRRA or power PWF hypothesis in terms of goodness-of-fit, neither do

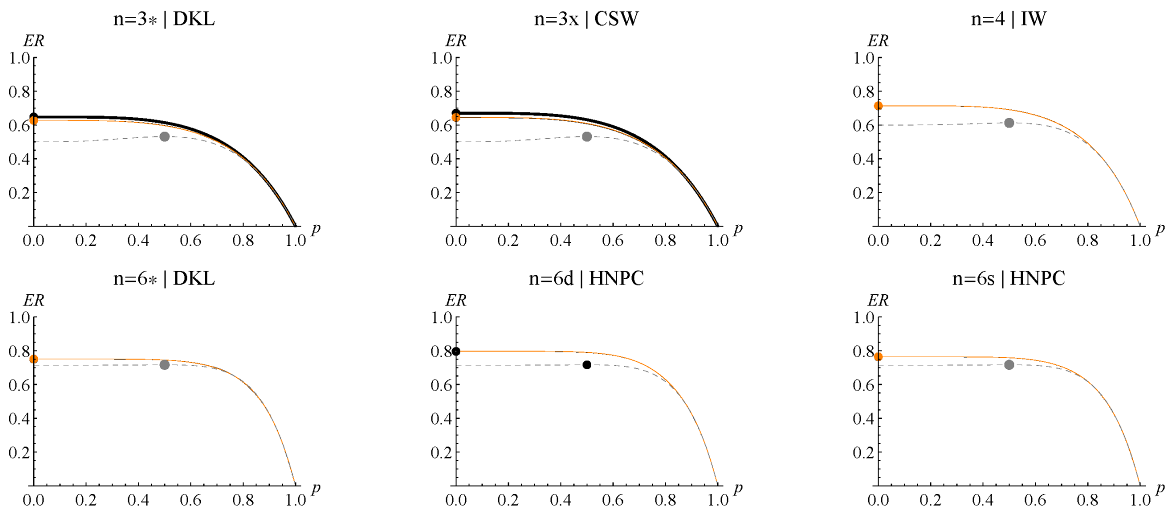

Figure 3: Expected Revenue vs. Reserve Price: Experimental Framework
Bold: UNI, Thin: BETA, Dashed: SBNE (c_μ=1), Orange: Prelec.

they lead to better revenue predictions or optimal reserve prices.

5 Conclusion

We analyze first-price auction data to assess the robustness of structural inferences (i.e., estimates, expected revenues and optimal reserve price recommendations) to the *quantity* and the *quality* of information available to the researcher. Our approach basically consists in assuming a parametric framework and in tracking changes in the researcher's inferences as the availability of different types of data changes.

We summarize our main findings as follows. First, structural inferences are sensitive to the *quantity* of information when its *quality* is low such as in field settings: using data on 'all bids' or on 'winning bids' yields significantly different estimates of the recovered distribution of values which contradict equilibrium predictions. Second, improving the *quality* of information in a field setting dampens the effect of information *quantity* and reveals out-of-equilibrium bidding patterns such as changing risk parameter estimates with *(i)* the *quality* of information, *(ii)* the number of bidders and *(iii)* the organization of the market. Third, a counterfactual analysis of expected revenues and optimal reserve prices suggests that the observed behavior is still best explains by the SBNE model with either CRRA preferences or a power form of probability perception. Our findings thus complement those of Bajari and Hortaçsu (2005) by assessing the robustness of Nash equilibrium predictions to out-of-equilibrium behavior. They also indicate a considerable sensitivity of structural inferences to the econometric procedures; our parametric risk estimates being at least half the size of the non-parametric ones of Bajari and Hortaçsu (2005). Clearly we remain cautious about extrapolating these findings to the analysis of real-world auctions as the field researcher typically has much less information about the auction fundamentals than in our Empirical or Hybrid frameworks. Yet, our study provides useful insights and highlights the importance of assessing the extent of misbehavior when using structural methods for policy making.

From an experimental standpoint, our analysis exploits an equivalence between the CRRA hypothesis and a power form of probability misperception to test the SBNE model with data from first-price auction experiments. Since both hypotheses imply linear bidding strategies when values are uniformly drawn, they are hard to disentangle in the usual context of symmetric auction experiments. Nonetheless, the data usually rejects the SBNE model with either assumption because of a nonlinear bidding behavior. We therefore considered two variants of probability misperception that capture the observed nonlinearity but neither of them significantly improves the model's goodness-of-fit, the seller's revenue predictions or the researcher's optimal reserve price recommendations. Finally, when compared to the inferences made from a field setting (i.e., the Empirical or Hybrid framework), these variants yield no markedly different revenue predictions or optimal reserve prices. To this extent, our study shows how structural estimates from a benchmark model may wildly differ with the type of information available to the researcher and still leave revenue predictions and optimal reserve prices pretty much unchanged.

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Appendix A: Proofs of propositions

For expository convenience, the following proofs assume values to be drawn from a distribution F with density f defined on the unit interval.

Proof of Proposition 1: Non-identification results from the fact that we are using a single first-order moment to estimate three parameters. The proof pertains to the WINBIDS scenario and follows the same steps for the ALLBIDS scenario. Consider $\theta_1^0 > 0$ and $\theta_2^0 > 0$ with $\theta_2^0 \neq 1$ and $c_\mu^0 = 1$. The model in (8) is first-order identified if the following expression

$$\int_0^v b_\mu(\vartheta, (\theta_1, \theta_2), c_\mu) \mathrm{d}F_\theta(\vartheta)^n = E(b_t^W | (\theta_1^0, \theta_2^0), 1) \tag{A-1}$$

has a unique solution $\theta_1 = \theta_1^0$, $\theta_2 = \theta_2^0$ and $c_\mu = 1$. The proof consists in showing that this is not the case. Assume $c_\mu = \theta_2 = 1$ and rewrite (A-1) so as to get

$$\frac{1}{1 + \frac{2n-1+1/\theta_1}{n\theta_1(n-1)}} = E(b_t^W | (\theta_1^0, \theta_2^0), 1). \tag{A-2}$$

Since the left-hand side of this equation is continuous on its domain and maps on the unit interval, a solution θ_1^* to (A-2) exists. Hence, the solution to (A-1) is not unique.

Proof of Proposition 2: Assume $\theta_2^0 = 1$ and consider any finite and positive k^0, θ_1^0 and c_μ^0 such that $\frac{\theta_1^0}{c_\mu^0} = k^0$. Then,

$$b_\mu(v, (\theta_1^0, \theta_2^0), c_\mu^0) = v - \frac{1}{v^{k^0(n-1)}} \int_0^v \vartheta^{k^0(n-1)} \mathrm{d}\vartheta \tag{A-3}$$

for all v. Note that θ_1^0 and c_μ^0 only enter (A-3) as a ratio k^0. Therefore, there exist infinitely many vectors of parameters $(\theta_1, \theta_2, c_\mu)$ such that $b_\mu(v, (\theta_1, \theta_2), c_\mu) = b_\mu(v, (\theta_1^0, \theta_2^0), c_\mu^0)$ for all v. This establishes that the model in (11) is not first-order identified. Now, assuming $c_\mu = 1$ we get from (7)

$$b_{it} = v_{it} - \int_0^{v_{it}} \left[\frac{F_\theta(\vartheta)}{F_\theta(v_{it})} \right] \mathrm{d}\vartheta + \varepsilon_{it}. \tag{A-4}$$

where θ_1 and θ_2 are identified conditional on v_{it}. In particular, if $\theta_2 = 1$ then (A-4) reduces to

$$b_{it} = \frac{n-1}{n-1+1/\theta_1} v_{it} + \varepsilon_{it} \tag{A-5}$$

Comparing (A-5) to $b_r(v, (1,1), c_r)$ or $b_\alpha(v, (1,1), c_\mu)$ we conclude that if $\hat{\theta}_1$ is an estimate of θ_1, then an estimate of c_μ is $c_r = 1/\hat{\theta}_1$ if one assumes homogeneous CRRA preferences or $c_\alpha = \hat{\theta}_1$ if one assumes homogeneous probability distortion.

Proof of claim about testing the null hypothesis in (14): We sketch the proof for the case of homogeneous CRRA preferences; the case of a homogeneous power PWF is analogous. The null hypothesis in (14), define a function $h : \mathbb{R} \to \mathbb{R}^2$ such that if the model holds and c_r^0 is the true risk parameter, then there is an implicit restriction on the parameter space of $\theta = (\theta_1, \theta_2)$ which is given by $\theta^0 = h(c_r^0)$. To test this implicit restriction, we consider the distance between an estimator $\hat{\theta}$ of θ^0 that is consistent under both the null and the alternative hypotheses and another estimator $\hat{\theta}^0$ that is consistent only under the null, and we test whether this distance is statistically significant.
A consistent estimator under the null hypothesis is given by $\hat{\theta}^0 = h(\hat{c}_r)$, where $\hat{c}_r$ is a consistent estimate of the true risk aversion parameter c_r^0. This consistent estimator can be found by estimating the linear model $b_{it} = \gamma v_{it} + \varepsilon_{it}$ by OLS and computing $\hat{c}_r = (n-1)(1-\hat{\gamma})/\hat{\gamma}$. Now, using $b_{it}(\theta) = v_{it} - \int_0^{v_{it}} \left[\frac{F_\theta(\vartheta)}{F_\theta(v_{it})}\right] \mathrm{d}\vartheta$, a consistent estimator $\hat{\theta}$ can be found by Nonlinear Least Squares, i.e.,

$$\hat{\theta} = \text{Arg Min}_\theta : \sum_{i,t} \left(b_{it} - b_{it}(\theta)\right)^2 , \tag{A-6}$$

Since it is asymptotically normal, we have $\sqrt{nT}(\hat{\theta} - \theta^0) \to N\left(0, J_0^{-1} I_0 J_0^{-1}\right)$, where I_0 is the asymptotic variance of the gradient of the objective function, and J_0 stands for the asymptotic Hessian evaluated at the true parameter value.
Following Gouriéroux and Monfort (1995), it can be shown that $\sqrt{nT}(\hat{\theta} - \hat{\theta}^0)$ is asymptotically normal with a covariance matrix given by $V = M_h J_0^{-1} I_0 J_0^{-1}$, where M_h is the orthogonal projection of $\partial h(c_r^0)$ on the space spanned by the columns of J_0. Notice that since the only effective restriction in (14) is the one affecting θ_2, the matrix M_h is singular. Therefore, an asymptotic test of (14) at the required confidence level can be computed using

$$nT(\hat{\theta} - \hat{\theta}^0) V^{-1} (\hat{\theta} - \hat{\theta}^0) \to \chi_1^2 \tag{A-7}$$

The squared root of a chi-squared distribution is a standard normal. On the other hand, the parameters of the Beta distribution must be positive, and accordingly, the Nonlinear Least Squares estimation is performed against the log of the parameters θ_1 and θ_2 rather than on θ_1 and θ_2 themselves. Therefore, the above test statistic can be simplified and reduced to a standard univariate significance test of the log of the second parameter θ_2 in the Beta distribution.

Appendix B: Estimated CDFs and SBNE bidding functions.

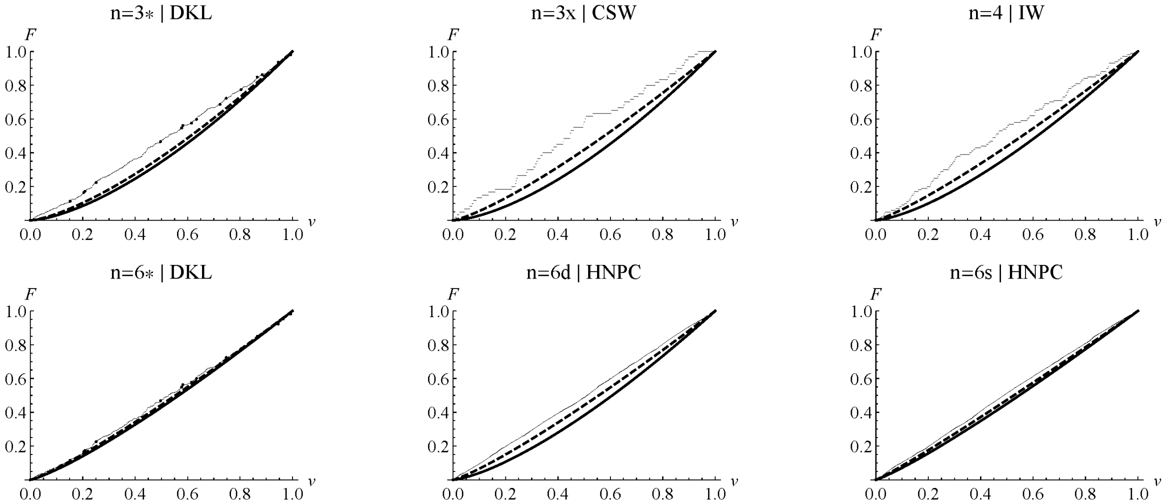

Figure B.1: Actual and estimated distributions of values: Empirical framework.
Plain: WINBIDS, Dashed: ALLBIDS, Dotted: Data

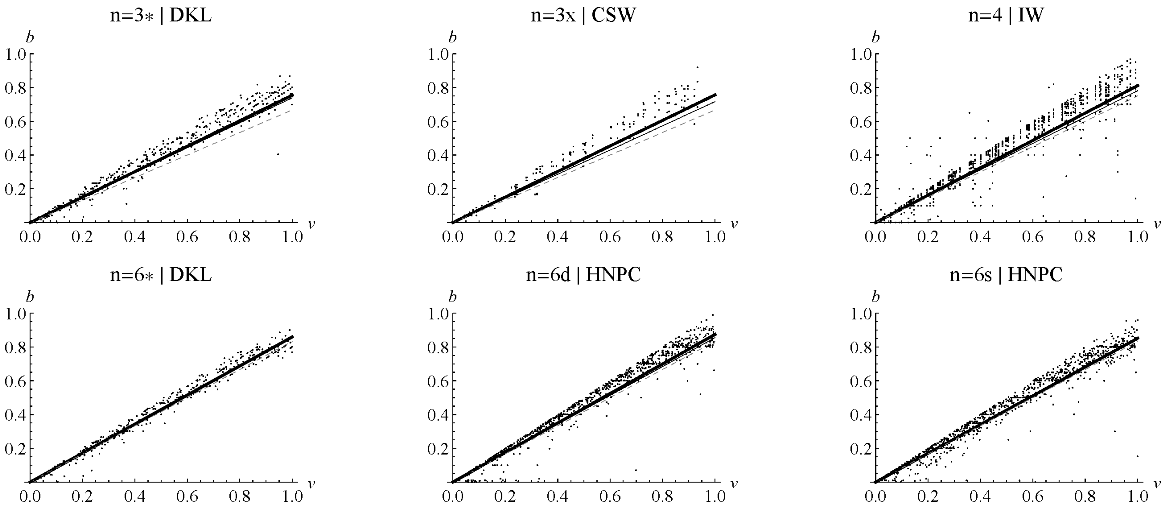

Figure B.2: Estimated and SBNE (c_μ=1) bidding functions: Empirical framework.
Thin: WINBIDS, Bold: ALLBIDS, Dashed: SBNE.

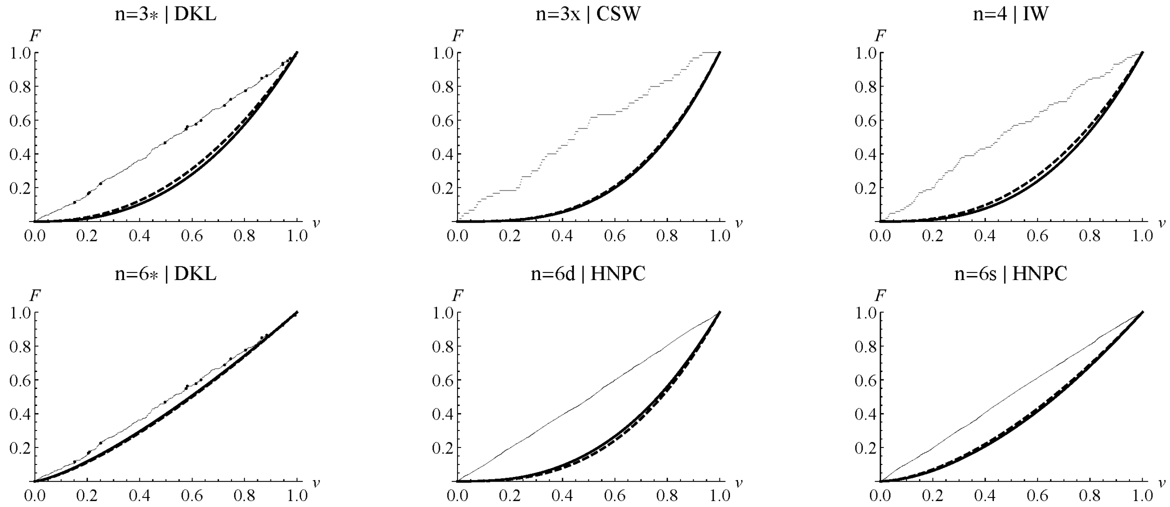

Figure B.3: Actual and estimated distributions of values: Hybrid framework.
Plain: WINBIDS, Dashed: ALLBIDS, Dotted: Data.

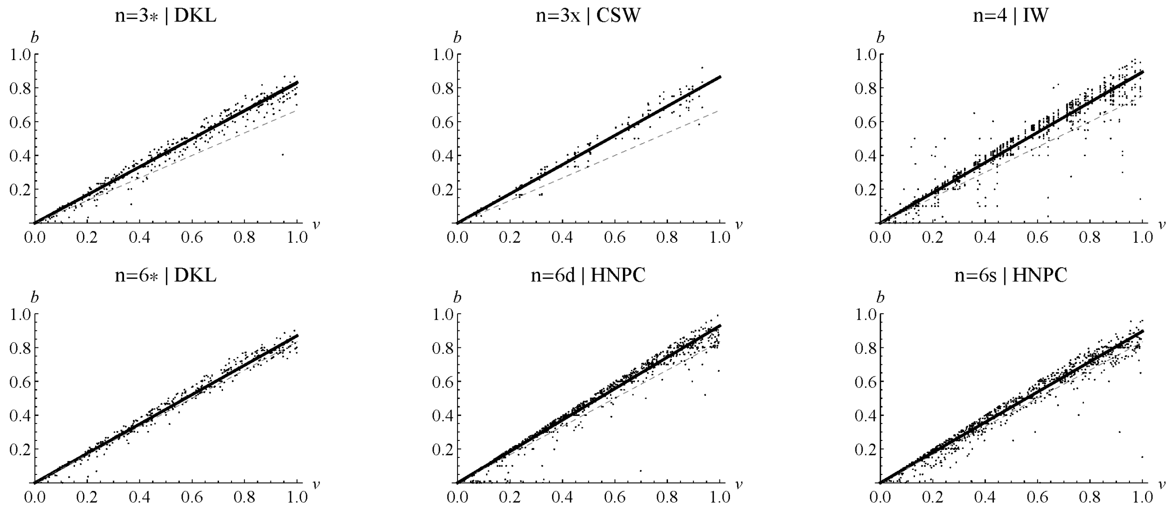

Figure B.4: Estimated and SBNE (c_μ=1) bidding functions: Hybrid framework.
Thin: WINBIDS, Bold: ALLBIDS, Dashed: SBNE.

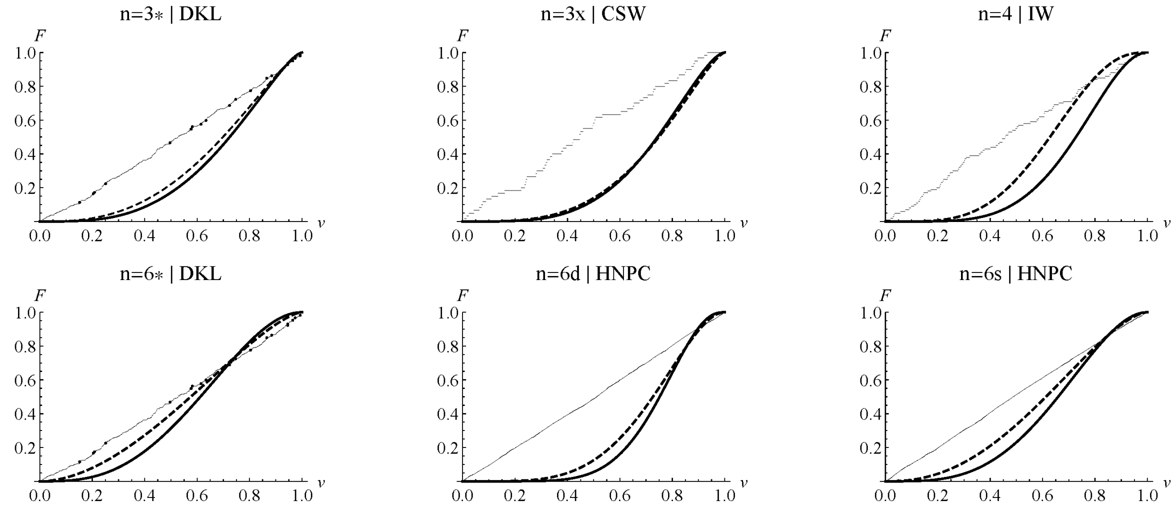

Figure B.5: Actual and estimated distributions of values: Experimental framework.
Plain: WINBIDS, Dashed: ALLBIDS, Dotted: Data sample

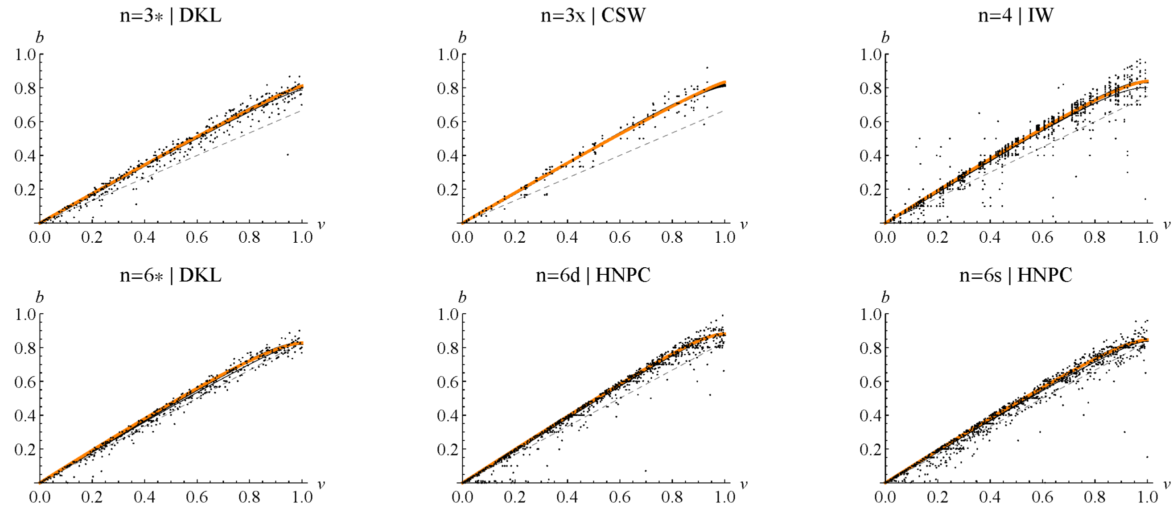

Figure B.6: Estimated and SBNE (c_μ=1 and θ_2=1) bidding functions: Experimental framework.
Thin: WINBIDS, Bold: ALLBIDS, Orange: Prelec, Dashed: SBNE.

Appendix C: Aggregate estimates for DKL / ALLBIDS.

This appendix reports the aggregate estimates for the DKL/ALLBIDS data when pooling the data for $n = 3$ and $n = 6$ bidders. As in the main analysis, we use the last 23 round of each session. Given individual $i = 1, \ldots, 18$ in round $t_s = 1, \ldots, 23$ of session $s = 1, 2, 3$, we minimize as usual the sum of squared differences between observed bids b_{it_s} and their expectations in equations (10) and (13) respectively.

Note that for each i and t_s, the variable n in these equations will be denoted by n_{it_s} with $n_{it_s} = 3$ if individual i in round t_s of session s is playing in a round with three bidders and $n_{it_s} = 6$ otherwise.

Table C.6: Estimation Outcomes for pooled DKL with 3 and 6 bidders.

Empirical			Hybrid			Experimental			
H_0^H	θ_{1A} [95% c.i.]	AIC [T]	H_0^H	θ_{1A} [95% c.i.]	AIC [T]	H_0^H	θ_{1A} [95% c.i.]	θ_{2A} [95%c.i.]	AIC [T]
.148	1.29 [1.21, 1.37]	-2.79 [828]	.000	2.01 [1.93, 2.09]	-6.30 [828]	.000	2.83 [2.58, 3.05]	1.64 [1.48, 1.78]	-6.40 [828]

Appendix D: Plots of expected revenues.

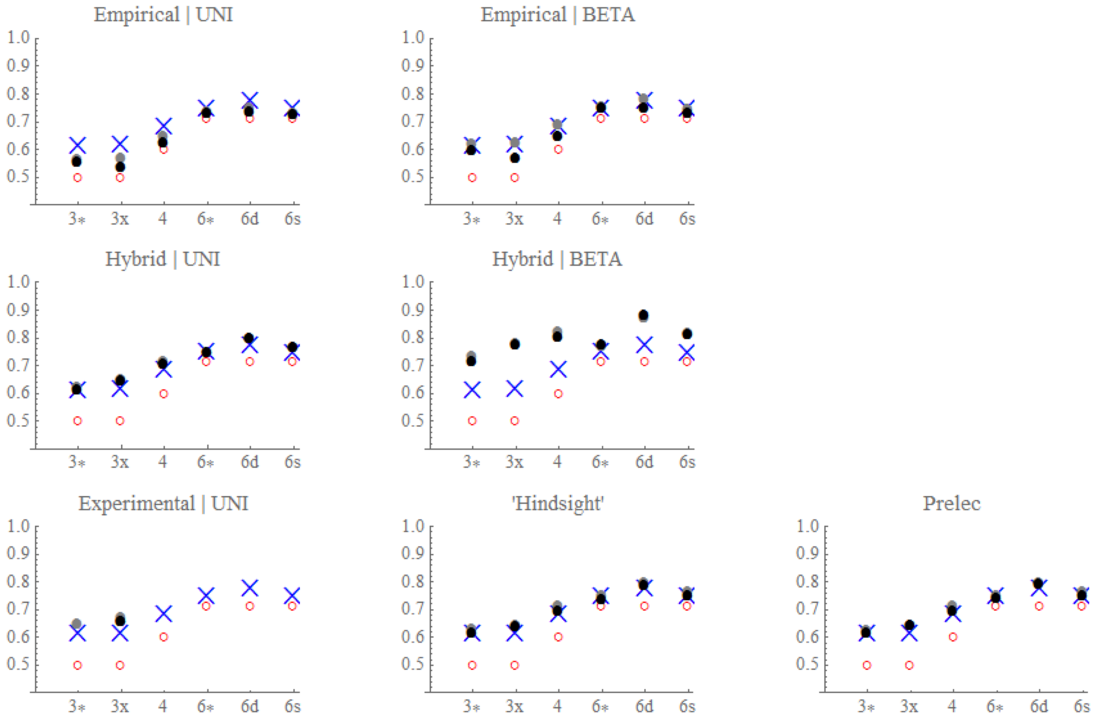

Figure D.1: Expected Revenues.
×: Observed, Black: ALLBIDS, Gray: WINBIDS, ∘: SBNE (c_μ=1).

Appendix E: Reduced form estimation outcomes of Prelec's PWF.

In this Appendix we report the Nonlinear Least Squares estimation outcomes of reduced form models of bidding that assume Prelec's (1998) exponential probability weighing function $\Phi(\pi) = e^{-\beta[-\ln \pi]^{\alpha}}$ with $\alpha,\beta > 0$. Using (1), we thus have

$$U(v_i, b_i) = u(v_i - b_i)\Phi(F\left(b_P^{-1}(b_i)\right)^{n-1}) \tag{A-8}$$

Differentiating this expression with respect to v_i, we get the following the following first-order nonlinear differential equation:

$$b'_P(v_i) = \frac{(n-1)\alpha\beta[v_i - b_P(v_i)][-\text{Log}(v_i^{n-1})]^{\alpha-1}}{v_i} \tag{A-9}$$

which has for boundary condition $b_P(p) = p$ and no closed-form solution for $\alpha \neq 1$ and $\beta \neq 1$. The seller's expected revenue is defined as:

$$\text{ER}_P((1,1),p,1) = \int_p^1 b_P(\vartheta,(\hat{\alpha},\hat{\beta}),p,1)\text{d}F_{(1,1)}(\vartheta)^n \tag{A-10}$$

Table E.1: Estimation Outcomes – Prelec's PWF.

Data	[n]	$H_0^{H^a}$	WINBIDS α_W [95% c.i.]	β_W [95% c.i.]	AIC[b] [T]	ALLBIDS $H_0^{H^a}$	α_A [95% c.i.]	β_A [95% c.i.]	AIC[b] [T]	$H_0^{\approx c}$
DKL	[3*]	0.007	1.17 [0.96, 1.37]	2.37 [2.09, 2.65]	-6.30 [54]	0.000	1.13 [1.05, 1.21]	2.11 [1.97, 2.24]	-6.30 [414]	0.235
CSW	[3x]	0.001	1.19 [0.80, 1.59]	2.87 [2.20, 3.55]	-5.75 [60]	0.001	1.09 [0.91, 1.26]	2.87 [2.40, 3.34]	-6.23 [180]	0.893
IW	[4]	0.004	1.57 [1.29, 1.86]	2.19 [1.89, 2.50]	-5.50 [250]	0.000	1.50 [1.34, 1.65]	1.68 [1.47, 1.90]	-5.31 [1000]	0.134
DKL	[6*]	0.003	1.63 [1.34, 1.92]	0.86 [0.71, 1.02]	-6.82 [69]	0.000	1.22 [1.15, 1.30]	1.01 [0.90, 1.11]	-6.87 [414]	0.002
HNPC	[6d]	0.000	1.83 [1.55, 2.11]	1.80 [1.49, 2.09]	-6.33 [150]	0.001	1.37 [1.25, 1.49]	1.84 [1.59, 2.10]	-6.25 [900]	0.010
HNPC	[6s]	0.002	1.59 [1.39, 1.79]	1.08 [0.92, 1.23]	-6.84 [150]	0.000	1.27 [1.18, 1.37]	1.12 [0.98, 1.26]	-6.21 [900]	0.011
HNPC*	[6d]	0.001	1.69 [1.17, 2.21]	1.76 [1.31, 2.21]	-6.28 [60]	0.000	1.22 [1.04, 1.39]	1.46 [1.12, 1.80]	-5.80 [360]	0.129
HNPC*	[6s]	0.006	1.42 [1.34, 1.49]	1.27 [.94, 1.59]	-5.37 [60]	0.000	1.14 [0.90, 1.34]	0.92 [0.63, 1.21]	-4.82 [360]	0.349

Note: [a]: Rejection probability of the null of homogeneity of the $(n \times S)$ $\tilde{\theta}_1$ and $\tilde{\theta}_2$-estimates; [b]: Akaike Information Criterium, $AIC = 2/T + ln(SSR/T)$ with T standing for the number of observations; [c]: Rejection probabilities of the null $\theta_{1W} = \theta_{1A}$ and $\theta_{2W} = \theta_{2A}$ according to $\chi^2(2)$ test.