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Demographic Transition and the Unobservable Scale Effects of Economic Growth

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Idea-based growth models usually predict that economic growth rates are increasing with the level or growth rate of the population. This scale effect prediction is intuitive and derived directly from the nonrivalry of ideas. However, time-series data over the last century generally did not support this scale effect prediction. This article illustrates why scale effects were unobservable. A modified idea-based model shows that economic growth rates increase with investments in human capital accumulation and population growth rates. The offsetting movements of these two factors during the demographic transition of the last century obscured the scale effects. (JEL E27 O40)

Keywords: Economic growth, scale effects, human capital, population, demographic transition

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I. Introduction

Idea-based models of economic growth usually predict scale effects of population growth, but this prediction receives little empirical support. The early idea-based models such as Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) predicted that an increase in the size of the population, other things equal, leads to a higher growth rate of per capita income. This prediction of scale effect of the level of population is inconsistent with the time-series evidence presented in Jones (1995b). Over the last century, total population and the people employed in the R&D sectors have exhibited significant, persistent upward movement, but the per capita income growth rates in industrialized economies show little to no persistent increases.

Many efforts have been taken to modify the idea-based models to eliminate this counterfactual scale effect prediction. However, the modified idea-based models trying to eliminate the scale effect of the level of the population still predicted a scale effect of the growth rate of population. As summarized by Jones (1999), the modified idea-based models of Jones (1995a), Kortum (1997), and Segerstrom (1998) predicted that the long-run growth rate of per capita income is proportional to the population growth rate; the models of Young (1998), Peretto (1998), Aghion et al. (1998), Dinopoulos and Thompson (1998) predicted that the growth rate of per capita income is proportional to the population growth rate and research effort in each sector. Thus, these models predict that as the rate of growth of population accelerates or retards, so does the rate of growth of per capita income.

The scale effect cannot be completely eliminated from the idea-based models because it is derived directly from the non-rivalry of ideas. The discovery of new ideas is the engine of economic growth in the idea-based models. Long-run economic growth is possible due to the increasing return derived from the non-rivalry of ideas. As emphasized by Romer (1986, 1990), ideas are non-rivalrous in the sense that the use of an idea by one person does not preclude, at the technological level, the simultaneous use of the idea by another person. More people means more ideas can be created, and therefore, more ideas can be used by each person. A higher growth rate of population, other things equal, should lead to a higher growth rate of ideas and then per capita income.

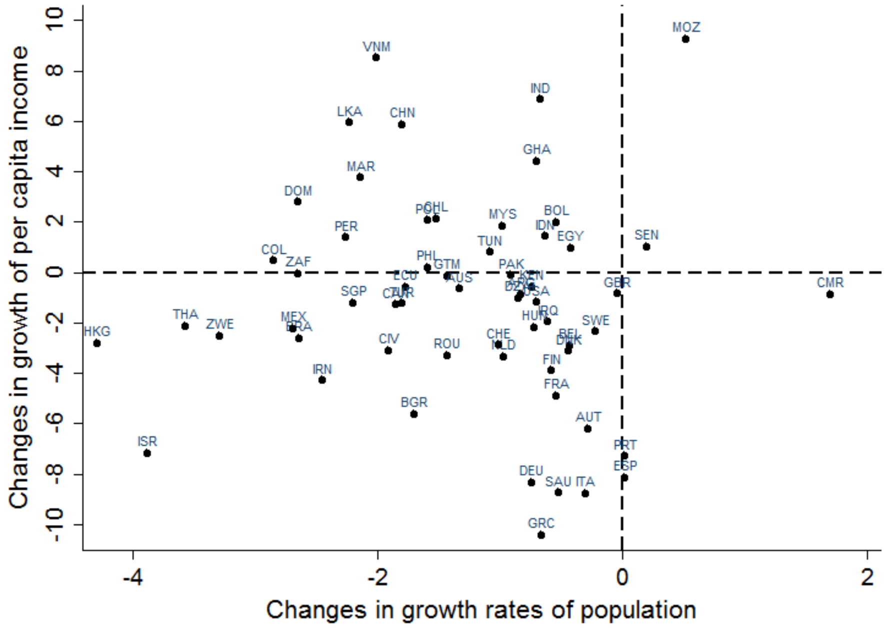

Figure 1: Changes in the growth rates of per capita income and population (1951-2015)
Note: Changes in growth rates are calculated as the different between 1951-1970 average and 1996-2015 average

However, empirical evidence on the scale effect of population growth rate is ambiguous. A stylized fact over the past half century and beyond was a demographic transition with dramatic declines in population growth rates. According to the scale effect prediction, we should observe significant declines in economic growth rates. As shown in Figure 1, almost all of the 60 sample countries experienced a decline in the growth rates of the population during 1950-2015, but a significant among of countries experienced an increase in the growth rates of per capita income during this period.[1] In addition, Table 1 summarizes the correlation between the growth rates of population and the growth rates of per capita income for each of the sample countries. We regress the growth rates of per capita income against the growth rates of the population for each country and then report the coefficient of

[1] See section IV for details of the data.

population growth rate in the table.[2] Among the 60 sample countries, only 22 report positive and statistically significant correlations between population growth rates and economic growth rates, other 38 countries report negative and significant correlations or insignificant correlation.

Table 1: The correlation between per capita income growth rates and population growth rates (1950-2015)

Significant and Positive		Insignificant		Significant and Negative	
Country	Coefficient	Country	Coefficient	Country	Coefficient
Switzerland	0.665***	United Kingdom	-0.449	Côte d'Ivoire	-0.856**
Australia	0.696**	Philippines	-0.488	Tunisia	-0.932*
Turkey	0.955**	Hungary	-0.506	Zimbabwe	-1.017*
Hong Kong	1.000***	Ghana	-0.515	Bulgaria	-1.175*
United States	1.113**	Morocco	-0.572	Kenya	-1.432***
Mexico	1.260***	Iran	-0.811	Saudi Arabia	-1.719***
Denmark	1.273*	Romania	-0.82	Algeria	-1.792***
Jordan	1.286***	Mozambique	-0.847	Sri Lanka	-1.995***
Israel	1.445***	Egypt	-0.904	Cameroon	-2.042***
Singapore	1.485***	Poland	-1.02	South Africa	-2.047***
Netherlands	1.622**	Argentina	-1.87	Pakistan	-2.188***
Germany	1.717***	Bolivia	-1.95	China	-2.731***
Canada	1.722***	Dominican Republic	0.026	Peru	-3.718***
Brazil	1.729***	Ecuador	0.04	Chile	-4.791***
Guatemala	2.253***	Colombia	0.223	Iraq	-5.809***
Sweden	2.778***	Senegal	0.259	India	-7.866***
France	4.086***	Malaysia	0.399		
Italy	4.759***	Thailand	0.554		
Finland	4.944***	Indonesia	0.911		
Belgium	5.160***	Austria	1.55		
Japan	5.794***	Portugal	2.15		
Greece	5.860**	Spain	2.3		

Note: We regress the growth rates of per capita income against population growth rates for each country and report the coefficient of the population growth rate in this table. Significance levels are *** p<0.01, ** p<0.05, * p<0.1.

[2] In the regression, the population data is delayed by ten years. Hence, the regression captures the correlation between current population growth rates and the growth rates of per capita income ten years later. It take about 20 years for a child to become an adult, so it is more likely that there is a lagged correlation between economic growth rates and population growth rates. Since population growth is partly due to the higher fertility and partly due to the lower mortality, we chose the lag of 10 years but not 20 years. We also tried the regressions with no population delay or delayed by 5, 15, or 20 years, and find the correlation is insensitive to these choices.

An interesting question is why the theoretically relevant scale effects do not generally appear in the empirical data. An intuitive hypothesis is that another determinant of the economic growth rates has been omitted from the idea-based models and the omitted variable moves in opposing direction to population growth rates. Human capital is one of the most likely omitted variables because human capital is usually treated as exogenous in the idea-based models, and the possibility of creating new ideas of an individual is likely to depend positively on his or her human capital. In addition, over the past half century and beyond, persistent increases in the level of education and dramatic declines in the population growth rate have been widely observed.

An idea-based model with endogenous human capital may shed some light on explaining the inconsistency between the theoretical prediction and the empirical evidence. The main argument of this article is that human capital should grow simultaneously with ideas over time. A critical fact is that even though the use of an idea by one person does not preclude the simultaneous use of the idea by another person, only the people with high enough human capital are able to use a specific idea. In other words, ideas will be productive only when it has been acquired by individuals in the form of human capital. Otherwise, it is equal to assume that providing cavemen with books documented the latest ideas of modern people will make the caveman as productive as modern people. In addition, the creating of new ideas usually require the inventor to acquire the cut edge ideas. It is quite less likely, for example, for an engineer to design a better machine but without maintaining the knowledge of the currently best machines.

Two definitions of human capital are usually used in the economic growth literature. The idea-based models such as Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) take the narrow definition and see human capital as years of education or training that a person received. Another broader definition of human capital is taken by theoretical models of growth based on unlimited human capital accumulation such as Lucas Jr (1988) and Becker, Murphy, and Tamura

(1990), in which human capital is measured by the productivity or ideas that a person gained through education or training. In this case, human capital is the combination of ideas with physical labors.

To allow human capital grows simultaneously with ideas, we prefer to take the broad definition and measure human capital as ideas embodied in physical labors. If we see human capital as years of education or training that a person received, it is impossible for human capital to grow simultaneously with ideas because the available time for a person to receive education and training is limited but the growth of ideas is unlimited. In addition, the learning efficiency of individuals should increase with the stock of ideas. Better ideas of production are always found to replace the old methods, and a person learned the latest ideas of production should have a higher human capital level as measured by productivity. Treating human capital as years of education is equal to assume that a person who received a 5 years education 1000 years ago had the same productivity as a person who received a 5 years education nowadays.[3]

To provide a potential explanation of why scale effects are unobservable, this article develops a modified endogenous growth model that allows the ideas and human capital to accumulate simultaneously over time. The model combines the basic element of the idea-based models - the non-rivalry of ideas - with the growth models that treat human capital as endogenous. The model depends heavily on the models of Romer (1990), Jones (1995a), and Becker, Murphy, and Tamura (1990), and keeps most of the assumptions of their models. The main implication of the model is that the growth rate of per capita income is proportional to the investment in education and the growth rate of population. The trade-off between "quality" and "quantity" of children made by utility optimizing parents result in the opposing movements of the growth rate of population and the level of education. The offsetting movements obscured the observation of the scale effects of population growth in the time-series data.

[3] It is worth to point out that since the idea-based models usually treat human capital as exogenous, there is no conceptual problem to see human capital as years of education or training a person received.

In the remaining of this paper, Section II summarize the growth models this paper depends on. Section III is the model and its implications. Section IV provides some empirical supports. Section V is the concluding remarks.

II. Literature

This section summarizes the growth models that this paper depends on and help to understand the distinction of the model provided in this paper. As illustrate in Jones (1999), the basic element of the early idea-based models such as the models of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) can be summarized by the following equations:

$$Y_t = \left(A_t L_{yt}\right)^{\alpha} K_t^{1-\alpha} \qquad (1)$$

$$\dot{A}_t = \delta L_{At} A_t \qquad (2)$$

The consumable good Y_t is produced using ideas A_t, labor L_{Yt}, and physical capital K_t. The production function is constant returns to labor and physical capital together, and increasing returns to ideas, labor and physical capital as a whole. New ideas, $\dot{A}_t$, are produced using only labor L_{At} and the existing stock of ideas A_t. In the steady-state, the growth rate of per capita income g_y equals the growth rate of ideas g_A and is proportional to the share of labor working in the R&D sector $s = L_{At}/L$ and the size of the population L:

$$g_y = g_A = g_k \equiv \delta s^* L \qquad (3)$$

These models predicted that a rise in the size of the population, other things equal, leads to proportional rising of growth rates. Jones (1995b) provided time-series evidence to reject this prediction of the scale effect of the level of population and argued that the counterfactual prediction resulted from the strong assumption that the growth rate of ideas is linear to the input in innovation.

The modified idea-based model of Jones (1995a), Kortum (1997), and Segerstrom (1998) relaxed this assumption by assuming that ideas are created according to:

$$\dot{A}_t = \delta L_{At}^{\lambda} A_t^{\phi} \tag{4}$$

where $\phi < 1$, and $0 < \gamma \leq 1$. In which, $\phi < 0$ corresponds to the case of "fishing out" and the rate of innovation decreases with the level of knowledge; $\phi > 0$ corresponds to the case of positive external returns. This modification eliminated the scale effect of the level of the population but predicted a scale effect of the growth rate of population. The modified idea-based models predicted that the growth rate of per capita income is proportional to the growth rate of population n:

$$g_y = g_A = g_k \equiv \frac{\lambda}{1-\phi} n \tag{5}$$

However, as shown in Table 1, significant declines in growth rates of the population does not necessarily result in significant declines in growth rates of per capita income. Since population and human capital are generally treated as exogenous in the idea-based models, the simple linear correlation between growth rates of population and growth rates of per capita income cannot be used to reject or support the scale effect predictions. This is because if there is another determinant of growth rates that has been omitted from the idea-based models and this factor moving opposite to population growth rates, the simple correlation between economic growth rates and population growth rates may be miss leading.

On the other hand, the growth models with endogenous human capital and fertility usually treat ideas as exogenous. As a result, scale effects of population growth are generally not predicted by these models. However, the offsetting movements of human capital and fertility are predicted in the model with endogenous human capital and fertility.[4] Here we depend on a simplified model of Becker,

[4] The trade-off between quality (human capital) and quantity (population growth rate) has been widely predicted by growth studies that are interested in the issues of demographic transition and the take-off of economic growth (Becker 1960, Becker

Murphy, and Tamura (1990) to illustrate the offsetting movements of human capital and fertility resulting from the utility-maximizing decision of rational parents. In an overlapping-generations economy in which everyone is identical and lives for two periods, childhood and adulthood. Each child consumes only a fixed quantity e of his parent's time and spend all his or her childhood accumulating human capital. Adults are endowed with T hours of working time and choose the number of offspring n and time spending on teaching each child z_t to maximize the dynastic utility function:

$$V_t = u(c_t) + a(n_t) n_t V_{t+1} \tag{6}$$

The dynastic utility of a parent V_t depends on his consumption c_t, the degree of altruism per child $a(n_t)$, the number of children n_t, and the utility of each child V_{t+1}. Diminishing marginal utility implies that the degree of parental altruism towards each child declines as the number of children increases, i.e. $a' < 0$. The consumable y_t is produced using only labor l_t and human capital h_t:

$$y_t = A l_t h_t \tag{7}$$

In which A measures the exogenous technology. Human capital is accumulated according to a learning technique with increasing returns to parents' human capital level:

$$h_{t+1} = B h_t^{\beta} z_t \tag{8}$$

In which $\beta \le 1$, and B measures the productivity of investments. Parents maximize dynastic utility function (6) subjecting to the time constraint:

$$T = l_t + n_t (e + z_t) \tag{9}$$

Due to the increasing returns to investing in human capital as the rising of parents' human capital, utility maximizing parents chose to have less number of children and invest more in each child.

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and Tomes 1976, Becker 1992, Hanushek 1992, Dahan and Tsiddon 1998, Becker, Murphy, and Tamura 1990, Morand 1999).

Depending on the assumption $\beta = 1$, the steady state growth rate of per capita income equals the growth rate of human capital and is proportional to the time invested in teaching each child:

$$1 + g^* = \frac{c_{t+1}}{c_t} = \frac{h_{t+1}}{h_t} = Bz^* \tag{10}$$

The scale effects of population growth are not shown up in this model because ideas are assumed as exogenous. However, the trade-off between human capital and the number of children implies that, once human capital and population are treated as endogenous in an idea-based model, there are potential offsetting movements in the human capital growth rate and population growth rate. In the following section, a modified growth model with endogenous ideas, human capital, and fertility shows this possibility.

III. A model with simultaneous growth of ideas and human capital

To show the consequence of the offsetting movements of human capital growth rates (g_h) and population growth rates (g_p) on the grow rates of per capita income (g_I), we combine the basic element of the idea-based models, i.e., the nonrivalry of ideas, with the model of Becker, Murphy, and Tamura (1990) to develop a model in which ideas and human capital growth simultaneously. We follow Becker, Murphy, and Tamura (1990) to assume that parents choose between investment in quantity and quality of children to maximize the dynastic utility. Increasing returns to human capital investment as the stock of human capital raises incentives parents to invest more in the education of each child and to choose a smaller number of children. As a result, g_h increases while g_p declines.

As in the idea-based models, the discovery of new ideas is the engine of economic growth. The difference is that now the human capital is endogenized. By assuming that the chance of creating new ideas depends positively on individuals' human capital level, the model predicts that the growth rates of the stock of idea is proportional to the growth rates of population and the growth rates of human capital. In addition, in the model, the growth rate of human capital is proportional to the investment in education. Consequently, the opposing movements of investments in the quality and quantity of children as a result of the utility maximizing choices of parents causing the unambiguous movements of the growth rates of ideas. Since the growth rates of per capita income equal the growth rates of ideas in the models, we simply cannot observe the theoretically relevant scale effects of population growth from the time-series data.

3.1. The model

The main claim of this article is that once we allow the human capital and ideas growth simultaneously in a model, we should predict that the scale effect of population growth rates is generally unobservable. The intuition behind this claim is that the growth rates of ideas should be proportional to the growth rates of population and the growth rates of human capital of individuals, the offsetting movements of fertility decline and education increase lead to ambiguous movements of the growth rates of per capital income. To make this claim, we construct an extremely simple economy to show the offsetting movements of the two determinants of economic growth rates.

Consider an overlapping-generations economy in which everyone lives for two periods, childhood and adulthood. People in the economy are divided into many groups according to their human capital and geographic regions. Individuals within a group are identical and share the same ideas and technology. Groups are different in adults' human capital level, technology level, and per capita income. For simplicity, assume that there is no technological spillover among groups, and it is impossible for people with different human capital to work in the same group due to different technologies adopted. Children are identical among groups, and each child consumes only a fixed quantity e of his parent's working time, and spends all his or her childhood accumulating human capital. Adults in group i work T hours and choose the number of offspring n_{it} and time spending on teaching each child z_{it} to maximize the dynastic utility.

The human capital of a child in the group i, $h_{i,t+1}$, depends on the average human capital of adults of the group $\bar{h}_{it}$, on his or her parent's human capital h_{it}, and on the time z_{it} a parent spends on teaching each child. The following simple learning technology is postulated:

$$h_{i,t+1} = \upsilon_i \bar{h}_{it}^{1-\gamma} \left(1 + h_{it} z_{it}\right)^{\gamma} \tag{11}$$

with $0 < \gamma < 1$. The group specific constant $\upsilon_i > 0$ measures the productivity of investments of the group i.[5] Here we assume each child is endowed with one unit of human capital, and if without education, his or her human capital in adulthood is $h_{i,t+1} = \upsilon_i \bar{h}_{it}^{1-\gamma}$. Human capital accumulation is decreasing return on parent's human capital h_{it} since $0 < \gamma < 1$, but constant return to the average adult human capital and parent's human capital as a whole. This assumption makes the permanent accumulation of human capital possible and also avoids increasing returns that will lead to accumulating infinite human capital within a finite time.

Two sources of external effects are captured by this learning technology. The first is a positive effect of the human capital of the teacher-parent on the learning of his or her children. The second externality is the positive effect of the group average human capital on the learning of all children. Since adults of the same group are identical, we have $\bar{h}_t = h_t$. In this study, we are interested in the growth of modern economics that with substantial individual human capital, so equation (11) can be simplified to

$$h_{i,t+1} \approx \upsilon_i h_{it} z_{it}^{\gamma} \tag{12}$$

[5] This learning technology is adapted from (Morand 1999). The difference is in the way social average human capital enters the learning function in order to avoid the increasing return to parent's human capital if parents are identical.

Assume that information within each group are complete, and that ideas discovered by an adult can be immediately learned by all other people within his/her group. Since the efforts of adults within a group are observable, each adult spends a constant share (s_i) of his/her working time on creating new ideas. The new ideas created will benefit the adults and their offspring by increasing the productivity.

New ideas created in each period $\dot{A}_i$ depends on the human capital of adults h_{it} , the number of adults N_{it} , the current technology level A_{it} , and the time spent on R&D $s_i T$:

$$\dot{A}_{it} = \delta_i H_{it}^{\gamma} A_{it}^{\phi} = \delta_i \left(h_{it} N_{it} s_i T \right)^{\gamma} A_{it}^{\phi} \tag{13}$$

with $\phi < 1$, and $0 < \gamma \leq 1$. In which, $\phi < 0$ corresponds to the case of "fishing out" and the rate of innovation decreases with the level of knowledge; $\phi > 0$ corresponds to the case of positive external returns. δ_i is a constant measure the group specific efficiency. $H_{it} = h_{it} N_{it} s_i T$ is the total human capital spent on R&D of the group i . This specification of the growth function of ideas is quite intuitive. Take the finding of the Newton's law of universal gravitation as an example, the chance for a group of people to find the Newton's law should be increasing with the number of people that is available to sit under trees, the human capital level of each person that enable them to think about the fall of apples more seriously, and the time each person available to sit under trees.

The critical different of the equation (13) from the production function of ideas in previous studies is that the human capital is endogenized. In addition, we apply the simplifying assumption that within each identical group the chance of creating a new idea depends on individuals' human capital level but independent of the working sectors.

The growth rate of ideas is

$$g_{itA} = \frac{\dot{A}_{it}}{A_{it}} = \delta_i \frac{H_{it}^{\gamma}}{A_{it}^{1-\phi}} \tag{14}$$

The stock of ideas can be inferred from its flows in the steady-state with a constant growth rate of ideas g_{iA}. The stock of ideas is

$$A_{it} = \left(\frac{1}{g_{iA}} \delta_i H_{it}^{\gamma} \right)^{\frac{1}{1-\phi}} = b_i \left(h_{it} N_{it} \right)^{\kappa} \tag{15}$$

$$b_i = \left(s_i T \right)^{\kappa} \left(g_{iA}^{-1} \delta_i \right)^{\kappa/\gamma}, \quad \kappa = \gamma / (1-\phi)$$

Each group produces the same kind of consumption good Y_{it} using ideas A_{it}, labor L_{it}, and physical capital K_{it}. Physical capital is accumulated consumer goods that do not wear out. We assume the single consumption good is produced according to a Cobb-Douglas production function in which ideas are "labor-augmenting":

$$Y_{it} = C_{it} + \Delta K_{it} = A_{it}^{\beta} L_{it}^{\beta} K_{it}^{1-\beta} \tag{16}$$

where C_{it} is the total consumption and ΔK_{it} is the net investment in physical capital. The level of human capital h_{it} does not show up in the production function because, as shown in the equation (15), it has been implicitly included in the A_{it}. Since we assume human capital increases proportionally to ideas, all ideas are embodied in physical labors in the form of human capital and can be used in the production function. In other words, the ideas used in production function is the usable ideas, and the simultaneous accumulation of human capital transfer all ideas into usable ideas. The assumption of $0 < \beta < 1$ implies the increasing returns to ideas. The production function can be measured in per capita term by dividing both sides by the number of adults (N_{it}):

$$Y_{it} / N_{it} = y_{it} = c_{it} + \Delta k_{it} = A_{it}^{\beta} l_{it}^{\beta} k_{it}^{1-\beta} \tag{17}$$

In which y_t is per capita output, l_t is the per capita time spent on production. We follow Becker, Murphy, and Tamura (1990) to assume that altruistic parents choose fertility and human capital investment for each child by maximizing the dynastic utility function:

$$V_{it} = u\left(c_{it}\right) + a\left(n_{it}\right) n_{it} V_{i,t+1} \quad (18)$$

The dynastic utility function is simplified with

$$u\left(c_{it}\right) = \frac{c_{it}^{\delta}}{\delta}, \quad a\left(n_{it}\right) = \alpha n_{it}^{-\varepsilon} \quad (19)$$

where $0 \le \varepsilon < 1$ and $0 < \sigma < 1$. The utility function is maximized subjecting to the following time and budget constraints:

$$\left(1-s\right)T = l_{it} + n_{it}\left(e + z_{it}\right) \quad (20)$$

$$c_{it} = b_i^{\beta}\left(h_{it}N_{it}\right)^{\kappa\beta}\left[\left(1-s\right)T - n_{it}e - n_{it}z_{it}\right]^{\beta} k_{it}^{1-\beta} - \Delta k_{it} \quad (21)$$

3.2. Equilibrium

In the following, we focus on analyzing a represent group i and omit the subscript i wherever there is no confusing. The arbitrage condition between per capita consumption in periods t and $t+1$ is

$$\frac{u'\left(c_t\right)}{au'\left(c_{t+1}\right)} = \alpha^{-1} n_t^{\varepsilon}\left(\frac{c_{t+1}}{c_t}\right)^{1-\delta} = R_{zt} = 1 + r_{zt} \quad (22)$$

Where r_{zt} is the rate of return on investments in human capital. To calculate the rate of return, we rewrite the Bellman equation using the learning technology (12), the time constraint (20), and the budget constraint (21) to yield

$$V_t\left(h_t\right) = \max\left\{\delta^{-1}\left[\left(bh_tN_t\right)^{\beta}\left[\left(1-s\right)T - n_te - n_t\left(\upsilon h_t\right)^{-1}h_{t+1}\right]^{\beta} k_t^{1-\beta} - \Delta k_t\right]^{\delta} + \alpha n_t^{1-\varepsilon} V_{t+1}\left(h_{t+1}\right)\right\}$$

(23)

Here we apply the simplification assumption of $\kappa = \gamma = 1$. Differentiating with respect to h_{t+1} and get:

$$-c_t^{\delta-1}\beta\left(bh_tN_t\right)^{\beta}\left(\upsilon h_t\right)^{-1} l_t^{\beta-1} k_t^{1-\beta} + \alpha n_t^{-\varepsilon} V_{t+1}' \le 0 \quad (24)$$

Using the envelope theorem provides:

$$V'_{t+1} = c_{t+1}^{\delta-1}\beta l_{t+1}^{\beta-1}k_{t+1}^{1-\beta}\left(bh_{t+1}N_{t+1}\right)^{\beta}\left(h_{t+1}\right)^{-1}\left(l_{t+1}+n_{t+1}z_{t+1}\right) \tag{25}$$

Combine (24) and (25) to get the arbitrage condition:

$$\alpha^{-1}n_t^{\varepsilon}\left(\frac{c_{t+1}}{c_t}\right)^{1-\delta} = \upsilon n_t\left(l_{t+1}+n_{t+1}z_{t+1}\right) \tag{26}$$

The rate of return depends positively on the productivity of investments in human capital υ. Since the rate of return measures the effect on c_{t+1} of increasing h_{t+1}, it also depends on the productivity of greater h_{t+1}, which depends on l_{t+1}, n_{t+1}, and z_{t+1}. In addition, according to the budget constraint $c_{t+1} = \left(bh_{t+1}n_tN_t\right)^{\beta} l_{t+1}^{\beta}k_{t+1}^{1-\beta} - \Delta k_{t+1}$, current period population growth rate has a positive effect on the next period consumption through increasing the stock of ideas.

By differentiating the utility function with respect to n_t and get the first-order condition for maximizing utility with respect to fertility:

$$\left(1-\varepsilon\right)\alpha n_t^{-\varepsilon}V_{t+1}\left(h_{t+1}\right) = \beta u'\left(c_t\right)b^{\beta}\left(h_tN_t\right)^{\kappa\beta} l_t^{\beta-1}k_t^{1-\beta}\left(e+z_t\right) \tag{27}$$

The marginal utility from an additional child is given on the left-hand side of the equation (27), while the left-hand side of the equation gives the total costs of producing and teaching a child. Costs depend on the fixed time (e) spend on each child, the endogenous time spend on teaching (z_t), and the productivity of labor. Parents from groups with the higher human capital and the larger population will choose to have a smaller number of children. In addition, the number of children chosen by the utility-maximizing parents is negatively correlated with the fixed cost of feeding each child and the endogenous costs of teaching each child.

The first order condition with respect to z is

$$\beta u'(c_t) A_t^{\beta} l_t^{\beta-1} k_t^{1-\beta} n_t = \alpha n_t^{1-\varepsilon} \gamma \upsilon h_t z_t^{\gamma-1} \frac{dV_{t+1}}{dh_{t+1}} \quad (28)$$

The economy converges to a steady-state growth path, with a constant time ($z*$) spent investing in each child's human capital, a constant fertility rate ($n*$), and a constant rate of growth over time in both h and c. In the steady-state equilibrium, the time spent investing in each child's human capital is

$$z* = \frac{\gamma\beta\delta\kappa e}{1-\varepsilon-\gamma\beta\delta\kappa} \quad (29)$$

The equilibrium education level of a child increases with learning efficiency γ, the share of human capital and ideas in the total output β, the elasticity of consumption δ, the efficiency of ideas creation κ, the fixed costs of raising a child, and the elasticity of altruism per child as their number increases ε.

The steady-state fertility is found by substituting into equations (22) and (26):

$$\alpha^{-1} n*^{\varepsilon-1} (1+g*)^{1-\delta} = \upsilon\left[(1-s)T - en*\right] \quad (30)$$

The equilibrium growth rate is

$$1+g* = \frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{A_{t+1}}{A_t} = \left(\upsilon z*^{\gamma}\, n*\right)^{\kappa} \quad (31)$$

$$g_c* = g_k* = g_A* = \kappa\left(g_h* + g_N*\right) = \kappa\left(\ln \upsilon z*^{\gamma} + \ln n*\right) \quad (32)$$

The equilibrium growth rate is proportional to the level of investment in children's learning $z*$ and the number of offspring of each parent $n*$. The ratio of k to Al is determined by the condition:

$$\upsilon n*\left[(1-s)T - en*\right] = R_h = R_k = \alpha^{-1} n*^{\varepsilon} (1+g*)^{1-\delta} \quad (33)$$

3.2. Population, education and economic growth

Theoretical studies that acknowledge the increasing returns resulting from the nonrivalry ideas generally predicted a positive effect of population growth on economic growth.[6] More recent idea-based models such as Jones (1995a), Kortum (1997), and Segerstrom (1998) stress the scale effects of population growth on economic growth. However, empirical studies generally do not support this prediction.

In this model, the long-run sustained per capita growth results from human capital growth, population growth, and increasing returns derived from the nonrivalry of ideas. Economic growth occurs not only because the economy is repeatedly discovering new ideas of production but also because individuals are continually acquiring these new ideas and transform them into human capital. More human capital leads to higher productivity in producing both consumption goods and new ideas. Therefore, the simultaneous growth of ideas and human capital, combined with the increasing returns associated with the nonrivalry ideas, delivers sustained long-run growth in per capita income.

During the transition dynamic, a decline in the population growth rate does not necessarily result in a decline in economic growth rate as long as the positive effect of the increasing in human capital is strong enough. As shown in the equation (27), the utility maximizing parents make a tradeoff between the number of children and the investment in each child. With the increase of human capital, according to the equation (11), the returns to the investments on human capital will increase. Hence, parents chose to have a smaller number of children in invest more on teaching each one. However, due to the differences in coefficients such as the elasticity of altruism per child ε, the share of human capital and ideas in the total output β, and the productivity of the idea creation b, per unit of education change results in different among of fertility changes in different groups and countries. The overall trends of economic growth depending on the relative changes in population growth rate and education

[6] Important studies including Phelps (1966), Nordhaus (1969), Judd (1985), Kremer (1993), Romer (1990), Jones (1995a), Kortum (1997), Segerstrom (1998), Grossman and Helpman (1991) and Aghion and Howitt (1992)

levels. This fact explains why the time-series evidence on the scale effect of population growth is ambiguous.

This model also consistent with several stylized facts in economic growth. First of all, the demographic transition can be explained by the model. Here demographic transition means, with the increase in per capita income, there is a long-run decline in population growth rate over time. This is because this model is based on the model of Becker, Murphy, and Tamura (1990) which has been designed to explain the demographic transition. The including of endogenous ideas in the model does not change the basic prediction of demographic transition.

More importantly, the model provides a quite intuitive explanation for the widely observed convergence of economic growth, i.e., the high-income countries tend to grow slower than low-income countries. In the model, the effect of human capital and the size of the population on economic growth is complementary. Holding one of these two elements constant, there is a decreasing return to the another element. The high-income countries are already with low population growth rates and high education levels, but the low-income countries usually with higher population growth rates and low education levels. The marginal returns to, for example, one year increase in education levels in high-income countries is lower than that in low-income countries. On the other hand, the population decline corresponding this one year increase in education will result in more economic growth rate decline in high-income countries than in the low-income countries. Consequently, convergence in the economic growth rate is derived from the model.

Long-run economic growth is possible in this model. As shown in the equation (32), for countries with a high learning efficiency (i.e., a large υ), as long as the equilibrium number of children each parent choose is not significantly less than one, a long run positive economic growth is possible. However, for countries with a low learning efficiency, a negative equilibrium growth rate of a population may imply a negative long-run economic recession. The model also implies that government policies have the potential to affect the long-run economic growth. A policy with the target of reducing the tradeoff between quality and quantity of children will result in higher long-run growth rates.

One important implication of the model is that long-run population growth rate will always decline as the increase of ideas and human capital level. If fertility declines to the extent that is lower than the replacement rate, the only way to keep an economic growth is through migration. Important welfare implications of migration and size of a country are evident from the models. If the long-run population growth is lower than the replacement rate, large countries will experience a longer growth period and accumulate more overall wealth, and per capita wealth will converge among countries. A country will always benefit from migrant from other countries no matter the wealth level attached of the immigrant. Ultimately, the world population will shrink while per capita wealth growth.

IV. Empirical evidence

Even though this model provides intuitive explanations for several stylized economic facts, rigorous test of the implications of the model by empirical data is impossible due to the data unavailability and model assumptions. In this simple model, we only consider the case that changes in the population only result from the changes in fertility. In the reality, mortality changes and migrations are also important sources of population changes. On the other hand, the human capital accumulation in our model is only a result of parents' investments and the positive externality of the social environment. But in the real world, a significant part of the education investment are made by governments. Hence, it is not a good idea to use the historical time-series data on population and education to calibrate and test the model.

Nevertheless, we can still apply some indirect empirical evidence to support the implications of the model. This section uses time-series data to show the tradeoff between the investments in human capital and the population growth rates. The empirical data also show that for a country that already with high incomes and high education levels, a decline in population growth rate will result in a decline in the economic growth rate. But for countries initially with low income and low education levels, a decline in population growth rate does not always lead to a decline in economic growth, because the positive effect of the increasing of investment in human capital neutralizes the negative effect of the population growth rate decline.

In the empirical study, the data for GDP and population come from the Conference Board Total Economy Database.[7] The country-level yearly data are available for the period of 1950-2015. The GDP is in 1990 US$ and converted at Geary Khamis PPPs. The population data is midyear population in thousands of persons. We calculate the yearly growth rates of the population and the per capita GDP from this dataset. The growth rates are calculated as 10-year moving average in order to eliminate short-term changes associated with business cycles or other transient disturbances. The data for education comes from International Human Development Indicators.[8] The years of schooling is measured by the average number of years of education received by people ages 25 and older. The data is available with a five-year interval from 1980-2005, and available for each year from 2006-2013. The yearly data from 1980-2005 is generated by a linear interpolation.

For the countries or regions (mentioned as countries for simplicity in the following) with available data for these three variables, we drop 9 low-income countries from the sample because this study does not consider the case of corner solution which is possible when per capita income and human capital level are quite low.[9] We also drop the 14 countries with extremely abundant natural resources per capita, such as Qatar, Kuwait, and the United Arab Emirates. High income in these countries is more likely result from abundant resources but not ideas. Finally, we get a sample of 69 economies.

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[7] Source: The Conference Board. 2015. The Conference Board Total Economy Database™, September 2015, http://www.conference-board.org/data/economydatabase/

[8] Source: Barro and Lee (2013), UNESCO Institute for Statistics (2013b) and HDRO estimates based on data on educational attainment from UNESCO Institute for Statistics (2013b) and on methodology from Barro and Lee (2013).

[9] We follow the classification of The World Bank in July 2011 to define that low-income economies are those that had per capita incomes of $1,005 or less in 2010.

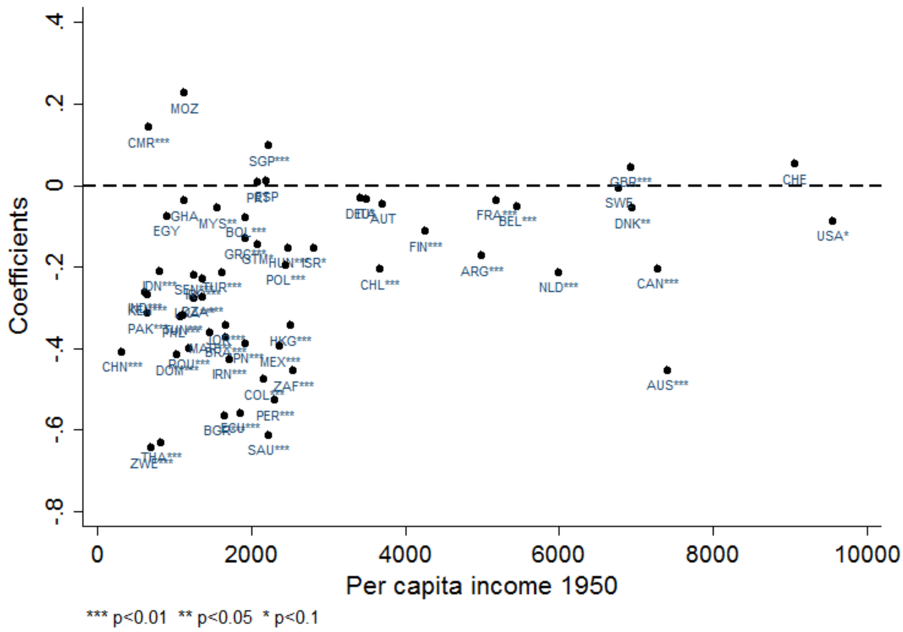

Figure 2: The correlation between population growth rates and years of schooling (1980-2015)
Note: Significance levels are *** p<0.01, ** p<0.05, * p<0.1

Figure 2 shows the tradeoff between education and population growth as predicted by the model. We regress the growth rates of population growth rates against the years of schooling for each country and then report the coefficient of the schooling in this Figure. Almost all countries show a significant and negative correlation between years of schooling and the growth rates of the population during the period of 1980-2015, and only 3 countries show the positive and significant relationship.[10] In addition, the countries with low initial income present larger tradeoff than the countries with high initial income. This evidence supports the basic argument of the model that with the rising of human capital levels, parents tend to invest more in the human capital of each child and choose to have less number of children.

[10] The education data for these countries are only available after 1980.

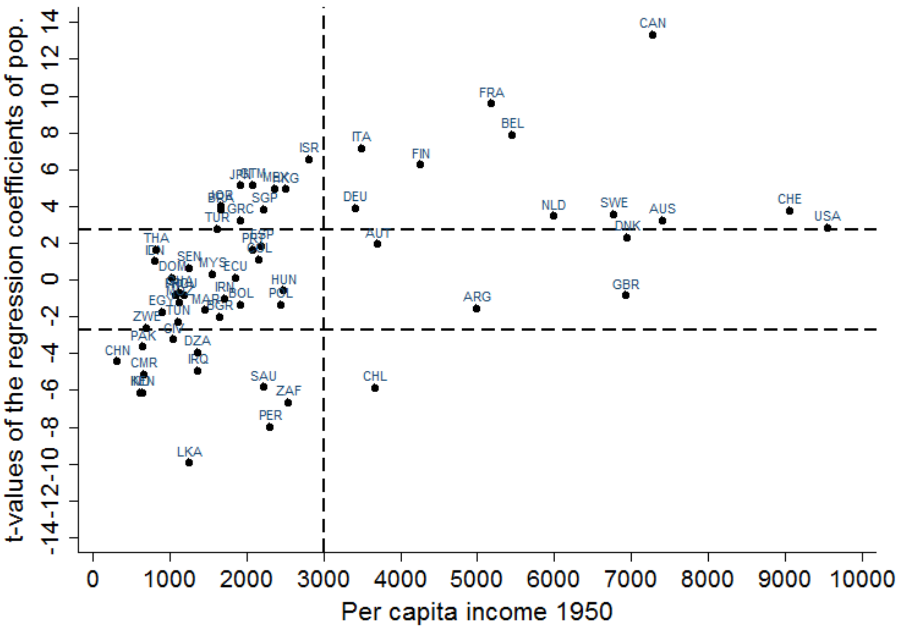

Figure 3: The correlation between the growth rates of per capita income and the growth rates of population

Figure 3 presents the correlation between the growth rates of per capita income and the growth rates of the population for countries with different initial per capita income. The reported value is the t-value of the coefficient of the population from the regression with per capita income as the dependent variable and with population growth rates as the independent variable. The two horizontal dotted line represent the t-value -2.7 and 2.7 respectively. Approximately, a t-value smaller than -2.7 or higher than 2.7 means the correlation is statistically significant. Most countries with an initial per capita income higher than 3000 US$ in 1950 present a significant and positive correlation between economic growth rates and population growth rates. However, for most of the countries with an initial income less than 3000 US$, the correlation between population growth rates and income growth rates in low-income countries are quite ambiguous.

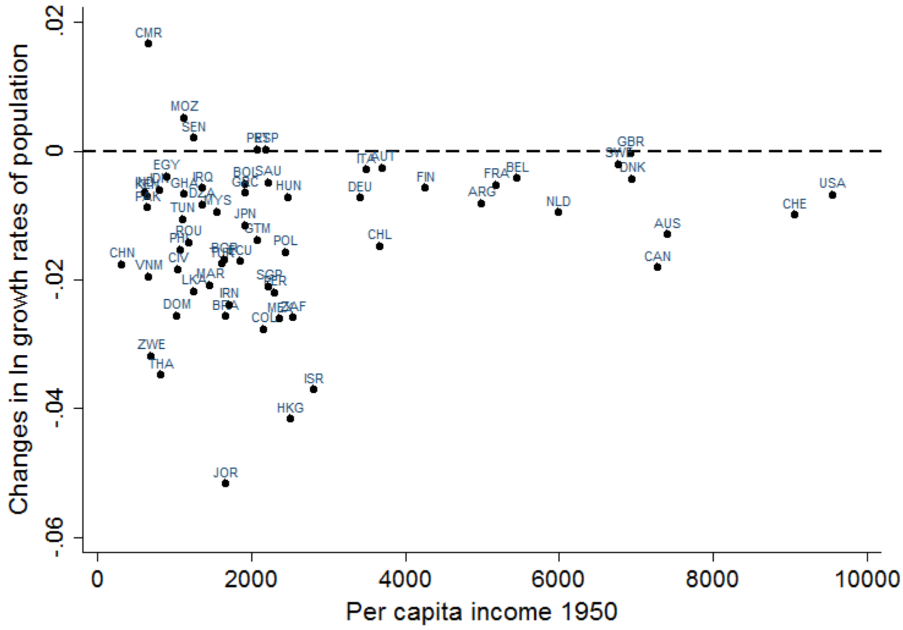

Figure 4: Changes in growth rates of population (1951-2015)
Note: Changes in growth rates are calculated as the different between 1951-1970 average and 1996-2015 average

These facts are consistent with the model predictions. As shown in Figure 4, all of the high-income countries and most of the low-income countries experienced a significant decline in the population growth rates. Hence, the Figure 3 implies that all high-income countries experienced a decline in the growth rates of per capita income, and the growth rates in low-income countries can be significantly increased, a significant decline or no significant change. The difference in the correlation between high and low-income countries can be explained by the differences in the marginal returns to increases in education and declines in the growth rates of population.

The difference in the marginal effects of changes in years of schooling is presented in Figure 5. According to the equation (32), the growth rates of per capita income is a function of the natural log of education investment. Hence, we plot the changes in the natural log of education against the per capita income in 1950 to show the difference in the effects of changes in education levels across countries with difference initial income. We find that the positive effects of change in education in high-income countries are much less that in low-income countries.

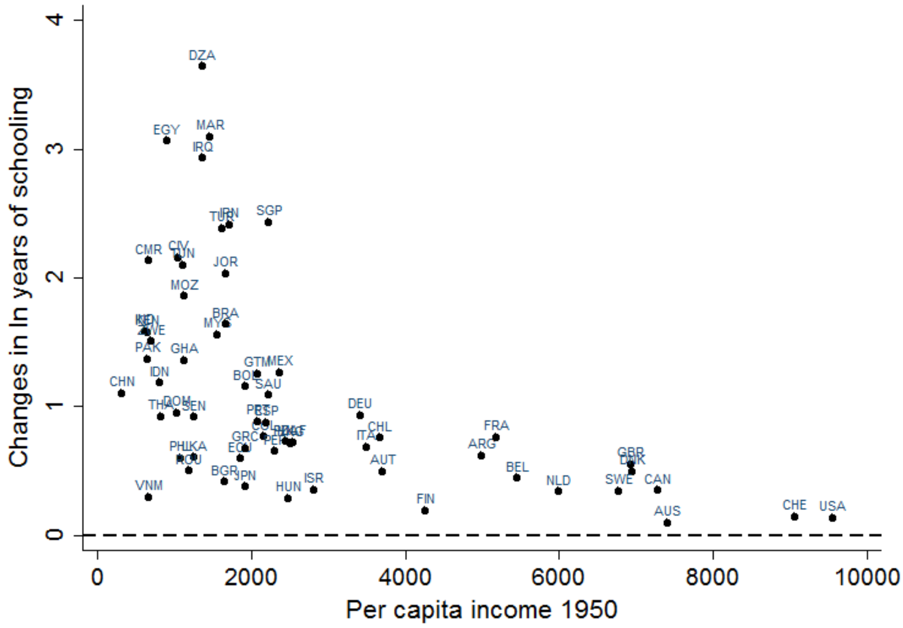

Figure 5: Changes in natural log of years of schooling (1951-2015)
Note: Changes are calculated as the different between 1951-1970 average and 1996-2015 average

The high-income countries have already with a high level of education and low level of population growth rate. The marginal positive effect of an increase in education is relatively small, and the marginal negative effect of a decline in the growth rate of population is relatively large. Consequently, the overall effect is dominated by the negative effect of population growth rate decline. For the low-income countries, the overall effect of increases in the investment in human capital accumulation and the declines in population growth rates are ambiguous. Even though the low-income countries experienced a significant decline in population growth rates, they also experienced a significant increase in the years of schooling. In addition, the marginal positive effect of the increase in education is relatively high in the low-income countries, and the marginal negative effect of the decline in population growth rates is relatively small. Consequently, it is possible for some of the low-income countries dominated by the positive effect of education increase while other low-income countries dominated by the negative effects of population growth rate decline.

V. Concluding Remarks

Building upon a number of earlier insights, this article shows that once we allow human capital and ideas are growing simultaneously in a model, we can provide an intuitive explanation of why the theoretically relevant scale effects of population growth are empirically unobservable. In the model, new ideas created in each period is an increasing function of the total among of human capital. The growth of per capital income can be accelerated by increasing in the size of the population or the level of human capital of each person. However, since both breeding and teaching children are time intensive, and the returns to the investment in human capital increases with the level of human capital, utility maximizing parents tend to choose a small number of children and to invest more in each child. Consequently, the trend of the economic growth rates depends on the relative strong of the positive effect of human capital increase and the negative effect of fertility decline.

The conclusion of this paper is in line with idea-based models such as Jones (1995a), Kortum (1997), and Segerstrom (1998) in that population growth is an important source of per capita income growth. A higher growth rate of population, other things equal, should lead to a higher growth rate of ideas and therefore per capita income. The difference is that in this model a decline in the population growth rate does not necessarily lead to a decline in the economic growth rate. Investments in human capital increase as the growth rates of population decline due to the trade-off between quality and quantity of children made by utility-maximizing parents. The positive effect of human capital accumulation tends to offset the negative effect of the declines in population growth rate.

This model is consistent with the model of Becker, Murphy, and Tamura (1990) in the prediction that the growth rate of per capita income is increasing with the investment in education and training.

Given population, a higher level of adults' human capital results in the higher growth of ideas and therefore per capita income. However, our model also shows that this positive effect of human capital accumulation may be unobservable in the time-series data. In this model, an increase in the investment in the accumulation of human capital should be always accompanied by a decline in the number of children each parent would like to have. Consequently, a decline in population growth rate caused by an increase in the investment in teaching children obscures the scale effect of investments in human capital.

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