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Monetary Policy and Indeterminacy after the 2001 Slump*

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Abstract

This paper estimates a New Keynesian model of the U.S. economy over the period following the 2001 slump, a period for which the adequacy of monetary policy is intensely debated. To relate to this debate, we consider alternative inflation series in the estimation. We find that only when measuring inflation with core PCE monetary policy appears to have been reasonable and sufficiently active to rule out indeterminacy. We then relax the assumption that inflation in the model is measured by a single indicator and re-formulate the artificial economy as a factor model where the theory's concept of inflation is the common factor to the empirical inflation series. We find that CPI and PCE provide better indicators of the latent concept while core PCE is less informative. Finally, we allow for positive trend inflation and the emerging results complement our previous findings. Again, even with these extensions, the only instance in which we can confidently rule out indeterminacy is when we measure inflation with core PCE.

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1 Introduction

It has become prevalent to think of monetary policy in terms of nominal interest rate feedback rules. In certain situations, for example, loose monetary policy, these rules may introduce indeterminacy and sunspot equilibria into otherwise stable economic environments. Lubik and Schorfheide (2004) and many others suggest that, empirically, such sunspots-based instability was confined to the seventies and that the post-Volcker years can ostensibly be characterized by determinacy. In light of this, the current paper examines determinacy over the years leading up to the Great Recession and we find that earlier claims cannot be sustained. In particular, indeterminacy can no longer be ruled out and this appears to be linked to loose monetary policy after the 2001 slump. We also establish that tests for indeterminacy are susceptible to the data used in the estimation.

The issue of loose monetary policy during the 2000s is closely related to Taylor (2007, 2012), who asserts that the Federal Reserve kept the policy rate too low for too long following the recession of 2001. He argues that this loose policy created an environment that ultimately brought the economy close to the brink. To bolster his thesis of an extra easy monetary policy, Taylor constructs an artificial path for the Federal Funds rate that follows his proposed rule. He characterizes this counterfactual rate's loose fitting to the actual rate as

> "[...] the biggest deviation, comparable to the turbulent 1970s." [Taylor, 2007, 2]

His view is disputed by many. Amongst them, Bernanke (2010) argues that Taylor's use of the headline consumer price index (CPI) to measure inflation in the Federal Reserve's reaction function is misleading. In fact, the Federal Reserve switched the inflation measures that inform its monetary policy deliberations several times over the last two decades. In particular, it moved away from the CPI to the personal consumption expenditure deflator (PCE) in early 2000. In turn, PCE was abandoned midway through 2004 in favor of the core PCE deflator (which excludes food and energy prices).[1] Bernanke (2015) revisits Taylor's exercise and constructs his own

[1] See Mehra and Sawhney (2010).

counterfactual Federal Funds rate using core PCE. Bernanke's verdict of the Federal Reserve's policy during the 2000s is inimical to Taylor's and he says that

> "[...] the predictions of my updated Taylor rule and actual Fed policy are generally quite close over the past two decades. In particular, it is no longer the case that the actual funds rate falls below the predictions of the rule in 2003-2005." [Bernanke, 2015]

Our paper sheds further light on this debate. It takes as a point of departure Taylor's claim of an analogy between the 1970s and the 2000s as well as one of the key recommendations for monetary policy that has emanated from New Keynesian modelling: interest rates should react strongly to inflation movements to not destabilize the economy. Phrased alternatively, if the central bank's response to inflation is tuned too passively in a Taylor rule sense, multiplicity and endogenous instability may arise. In fact, the U.S. economy of the 1970s can be well represented by an indeterminate version of the New Keynesian model as was shown by Lubik and Schorfheide (2004). Along these lines, the current paper turns Taylor's *too low for too long* story into questioning whether the Federal Reserve operated on the indeterminacy side of the rule after the 2001 slump.

The empirical plausibility of a link between monetary policy and macroeconomic instability was first established by Clarida, Gali and Gertler (2000). They estimate variants of the Taylor rule and their research suggests that the Federal Reserve's policy may have steered the economy into an indeterminate equilibrium during the 1970s. Yet, they also find that the changes to policy which have taken place after 1980 – essentially a more aggressive response to inflation – brought about a stable and determinate environment. Lubik and Schorfheide (2004) reinforce this point but they refrain from using a single equation approach. They recognize that indeterminacy is a property of a rational expectations system and apply Bayesian estimation techniques to a general equilibrium model. Their results parallel the earlier findings that the U.S. economy veered from indeterminacy to determinacy around 1980 – largely as the result of a more aggressive response of monetary policy towards inflation.

Moreover, this monetary policy change had perhaps an even greater influence on the economy: the transformation from the Great Inflation of the 1970s to the

Great Moderation is often conjoined to the conduct of monetary policy.[2] Yet, the Great Moderation came to an end sometime during the 2000s, and it was followed by enormous economic volatility. Our aim is to examine the possible connection between this transformation and an alteration in the Federal Reserve's monetary policy. In particular, we concentrate on the effects of a possibly too easy monetary policy after the 2001 slump. We frame our analysis from the perspective of the (in-)determinacy debate and conduct it under the umbrella of the Bernanke versus Taylor dispute by considering the measures of inflation that repeatedly occur in the discussion: CPI, PCE and core PCE.

Accordingly, we estimate a small-scale New Keynesian model allowing for indeterminacy over the period between the 2001 slump and the onset of the Great Recession, thus, the NBER-dated 2002:I-2007:III window to be precise. To test for indeterminacy, we employ the method of Lubik and Schorfheide (2004). This approach enables us to compute the posterior probabilities of determinacy and indeterminacy. Given the shortness of our period of interest, we take as starting point the same basic New Keynesian model, priors and observables as Lubik and Schorfheide (2004). This allows us to create a continuity between their and our results.

We establish a number of new insights regarding recent U.S. central bank policy. For example, we can indeed expose a violation of the Taylor principle for most of the 2000s when using CPI to measure inflation. This finding supports the visual inspection checks based on single equations in Taylor (2012) who coined the phrase *Great Deviation* to refer to this period. Hence, the 2002:I to 2007:III period would appear to be best described by an indeterminate version of the New Keynesian model. Our upshot is different when basing the analysis on PCE data: we can neither rule in nor rule out indeterminacy. Finally, the evidence in favor of indeterminacy altogether vanishes when we use core PCE. Monetary policy then appears to have been quite appropriate. This conclusion parallels the insight from Bernanke's (2015) counterfactual Federal Funds rate.

We next consider whether our results are an artifact of the six year sample of data. To address this issue, we re-estimate the model on rolling windows of fixed length (23

[2] See, for example, Benati and Surico (2009), Bernanke (2012), Coibion and Gorodnichenko (2011), Arias, Ascari, Branzoli and Castelnuovo (2014) and Hirose, Kurozumi and Van Zandweghe (2015).

quarters to match the length of the 2002:I-2007:III period) starting in the mid-1960s and focussing on the same inflation measure as Lubik and Schorfheide (2004) namely CPI inflation. The outcomes of the indeterminacy test performed on rolling windows are highly plausible. In particular, we identify only two broad periods (i.e. several consecutive windows) in which a passive policy has likely led to indeterminacy: the 1970s and the post-2001 period. The first period, which coincides with the span of the Burns and Miller chairmanships, exactly matches the indeterminacy duration, as well as the timing of the switch to determinacy in 1980, that Coibion and Gorodnichenko (2011) document. We take this analogy as a reassuring validation of our small sample approach, i.e. even though our period of interest is quite short, it is possible to infer meaningful information from it.[3]

We then attend the issue of how best to measure inflation in the New Keynesian model. We address the ambiguity between the theoretical concept and the empirical inflation proxies by employing the DSGE-factor model methodology proposed by Boivin and Giannoni (2006). Accordingly, we combine various measures of inflation in the measurement equation and re-estimate our model. CPI and PCE emerge as better indicators of the concept of inflation than core PCE. Moreover, indeterminacy cannot be ruled out.

Finally, we allow for positive trend inflation in the artificial economy. As shown by Ascari and Ropele (2009) and others, trend inflation affects the determinacy region in the New Keynesian model. Nevertheless, we are still not able to close out indeterminacy.

Perhaps most closely related to our work are Belongia and Ireland (2015) who, like us, evaluate the Federal Reserve's monetary policy during the 2000s.[4] Belongia and Ireland (2015) estimate a time-varying VAR to track the evolution of Federal Reserve policy that occurred through the 2000s. They find evidence of a change in the Federal Reserve's behavior away from stabilizing inflation towards stabilizing output and also of persistent deviations from the estimated policy rule. While similar in spirit to our results they do not address issues of indeterminacy.

[3] Judd and Rudebusch (1998) is another example of an evaluation of monetary policy over similarly short sample periods.

[4] See Fackler and McMillin (2015), Fitwi, Hein and Mercer (2015), Groshenny (2013) and Jung and Katayama (2014) for related exercises.

Bianchi (2013) examines the Federal Reserve's policy post-WWII taking a Markov switching rational expectations approach with two monetary policy regimes (i.e. *Hawk* and *Dove*). Bianchi characterizes monetary policy in the early 2000s as Hawkish and identifies a switch to a Dove regime after 2005. His approach to deal with the issue of passive monetary policy is by requiring a linear representation of the Markov switching model to have a unique solution. Phrased alternatively, the regime transitions do not imply moving from determinacy to indeterminacy as both regimes are determinate. Hence, Bianchi's model cannot address questions involving sunspot equilibria as in our paper.

The remainder of the paper evolves as follows. The next section sketches the model and its solution. Section 3 presents the econometric strategy and baseline results. Robustness checks are conducted in Section 4. Section 5 relaxes the assumption that model inflation is properly measured by a single empirical indicator. In Section 6 we consider models with trend-inflation. Section 7 concludes.

2 Baseline model

The familiar three linearized equations summarize our basic New Keynesian model:

$$y_t = E_t y_{t+1} - \tau(R_t - E_t \pi_{t+1}) + g_t \qquad \tau > 0 \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(y_t - z_t) \qquad \kappa > 0,\ 0 < \beta < 1 \tag{2}$$

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)(\psi_\pi \pi_t + \psi_y [y_t - z_t]) + \epsilon_{R,t} \qquad 0 \leq \rho_R < 1. \tag{3}$$

Here y_t stands for output, R_t denotes the nominal interest rate and π_t symbolizes inflation. E_t represents the expectations operator. Equation (1) is the dynamic IS relation reflecting an Euler equation. Equation (2) describes the expectational Phillips curve. Finally, equation (3) represents monetary policy, i.e. a Taylor-type rule in which $\psi_\pi > 0$ and $\psi_y > 0$ are chosen by the central bank and echo its responsiveness to inflation and the output gap, $y_t - z_t$. The term $\epsilon_{R,t}$ denotes an exogenous monetary policy shock whose standard deviation is given by σ_R. The other fundamental disturbances involve exogenous shifts of the Euler equation which are captured by the process g_t and shifts of the marginal costs of production captured by z_t. Both variables follow AR(1) processes:

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t} \qquad 0 < \rho_g < 1$$

and

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t} \qquad 0 < \rho_z < 1.$$

We denote by σ_g and σ_z the standard deviations of the innovations $\epsilon_{g,t}$ and $\epsilon_{z,t}$. Finally, the term $\rho_{g,z}$ denotes the correlation between the demand and supply innovations. Then, the vector of model parameters entails

$$\theta \equiv \left[\psi_\pi, \psi_y, \rho_R, \beta, \kappa, \tau, \rho_g, \rho_z, \rho_{g,z}, \sigma_R, \sigma_g, \sigma_z\right]'.$$

Indeterminacy implies that fluctuations in economic activity can be driven by arbitrary, self-fulfilling changes in people's expectations (i.e. sunspots). Concretely, in our simple New Keynesian model, indeterminacy occurs when the central bank passively responds to inflation changes, i.e. when $\psi_\pi < 1 - \psi_y (1-\beta)/\kappa$.

To solve the model, we apply the method proposed by Lubik and Schorfheide (2003) in which case the full set of rational expectations solutions takes on the form

$$s_t = \Phi(\theta) s_{t-1} + \Phi_\varepsilon(\theta, \widetilde{\mathbf{M}})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t \qquad (4)$$

where s_t is a vector of model variables,

$$s_t \equiv \left[y_t, R_t, \pi_t, E_t y_{t+1}, E_t \pi_{t+1}, g_t, z_t\right]',$$

ε_t denotes a vector of fundamental shocks and ζ_t is a non-fundamental sunspot shock.[5] The coefficient matrices $\Phi(\theta)$, $\Phi_\varepsilon(\theta, \widetilde{\mathbf{M}})$ and $\Phi_\zeta(\theta)$ are related to the structural parameters of the model. The sunspot shock satisfies $\zeta_t \sim i.i.d.\mathsf{N}(0, \sigma_\zeta^2)$. Indeterminacy can manifest itself in two ways: (i) through pure extrinsic non-fundamental shocks, ζ_t (a.k.a sunspots), disturbing the economy and (ii) it may affect the propagation mechanism of fundamental shocks through $\widetilde{\mathbf{M}}$.

3 Estimation and Baseline Results

This section describes the data as well as the estimation strategy. It is followed by a presentation and discussion of our baseline results.

[5] Under determinacy, the solution (4) boils down to $s_t = \Phi^D(\theta) s_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$.

Table 1 - Prior and posteriors of DSGE parameters.

			Priors	Posterior Mean [5th pct, 95th pct]	
Name	Range	Density	Prior Mean (Std. Dev.)	CPI Indeterminacy	Core PCE Determinacy
ψ_π	$\mathbb{R}^+$	Gamma	1.10 (0.50)	0.84 [0.61,0.98]	3.01 [1.97,4.17]
ψ_y	$\mathbb{R}^+$	Gamma	0.25 (0.15)	0.19 [0.05,0.41]	0.28 [0.07,0.64]
ρ_R	[0,1)	Beta	0.50 (0.20)	0.83 [0.74,0.90]	0.76 [0.64,0.85]
π^*	$\mathbb{R}^+$	Gamma	4.00 (2.00)	3.28 [1.27,6.01]	1.99 [1.67,2.31]
r^*	$\mathbb{R}^+$	Gamma	2.00 (1.00)	1.15 [0.47,2.01]	1.40 [0.84,2.01]
κ	$\mathbb{R}^+$	Gamma	0.50 (0.20)	0.91 [0.51,1.41]	0.71 [0.31,1.19]
τ^{-1}	$\mathbb{R}^+$	Gamma	2.00 (0.50)	1.66 [1.00,2.49]	1.62 [0.95,2.48]
ρ_g	[0,1)	Beta	0.70 (0.10)	0.60 [0.45,0.73]	0.80 [0.72,0.87]
ρ_z	[0,1)	Beta	0.70 (0.10)	0.80 [0.68,0.89]	0.61 [0.49,0.74]
ρ_{gz}	[-1,1]	Normal	0.00 (0.40)	-0.28 [-0.72,0.17]	0.86 [0.57,0.97]
$M_{R\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	-0.57 [-1.90,1.00]	
$M_{g\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	-1.99 [-2.92,-1.05]	
$M_{z\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	0.41 [0.05,0.83]	
σ_R	$\mathbb{R}^+$	IG	0.31 (0.16)	0.16 [0.12,0.21]	0.16 [0.12,0.21]
σ_g	$\mathbb{R}^+$	IG	0.38 (0.20)	0.28 [0.18,0.40]	0.19 [0.14,0.25]
σ_z	$\mathbb{R}^+$	IG	1.00 (0.52)	0.74 [0.54,1.03]	0.62 [0.47,0.82]
σ_ζ	$\mathbb{R}^+$	IG	0.25 (0.13)	0.20 [0.12,0.30]	

Notes: The inverse gamma priors are of the form $p\left(\sigma|\upsilon,s\right) \propto \sigma^{-\upsilon-1}e^{-\frac{\upsilon s^2}{2\sigma^2}}$, where $\nu = 4$ and s equals $0.25, 0.3, 0.6$ and 0.2, respectively. The prior for ρ_{gz} is truncated to ensure that the correlation lies between -1 and 1. The prior predictive probability is 0.527.

3.1 Data and priors

We employ Bayesian techniques for estimating the parameters of the model and test for indeterminacy using posterior model probabilities. The measurement equation relating the elements of s_t to the three observables, x_t, is given by

$$x_t = \begin{bmatrix} 0 \\ \pi \\ r + \pi \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 \end{bmatrix} s_t \tag{5}$$

where π and r are annualized steady state inflation and annualized steady state real interest rates. Equation (4) and (5) provide a state-space representation for the linearized model that allows us to apply standard Bayesian estimation techniques. The technical appendix provides further details.

We use HP-filtered per capita real GDP and the Federal Funds Rate as our observable for output and the nominal interest rate. These choices make our baseline empirical analysis comparable to Lubik and Schorfheide (2004) as we want to understand the effect of our small sample. To draw up our analysis in the Bernanke versus Taylor debate, we consider in turn three different measures of inflation: CPI, PCE deflator and core PCE (annualized percentage changes from the previous quarter). The data covers the period between the 2001 slump and the onset of the Great Recession, i.e. 2002:I to 2007:III.

Our baseline priors are identical to the ones in Lubik and Schorfheide (2004) and they are reported in Table 1. Following Lubik and Schorfheide (2004) we replace $\widetilde{\mathbf{M}}$ in equation (4) with $\mathbf{M}^*(\theta) + \mathbf{M}$ where $\mathbf{M} \equiv [M_{R\zeta}, M_{g\zeta}, M_{z\zeta}]'$. We select $\mathbf{M}^*(\theta)$ such that the responses of the endogenous variables to fundamental shocks are continuous at the boundary between the determinacy and the indeterminacy regions. We set the prior mean for $\mathbf{M}$ equal to zero.

3.2 Testing for indeterminacy

For each measure of inflation, we estimate the model over the two different regions of the parameter space, i.e. determinacy and indeterminacy. To assess the quality of the model's fit to the data we present marginal data densities and posterior model probabilities for both parametric zones. We approximate the data densities using Geweke's (1999) modified harmonic mean estimator. Table 2 reports our results.

Following Lubik and Schorfheide (2004) and Taylor (2007, 2012), we begin by using headline CPI to measure inflation. In this case, the data favors the indeterminate model: the posterior probability of indeterminacy is 0.90. This result suggests that

Taylor's characterization of the Federal Reserve's monetary policy as *too low for too long* is in fact consistent with indeterminacy and potentially has veered the economy into instability.

Table 2: Determinacy versus Indeterminacy

	Log-data density		Probability	
Inflation measure	Determinacy	Indeterminacy	Determinacy	Indeterminacy
CPI	-95.48	-93.28	0.10	0.90
PCE	-85.42	-85.75	0.58	0.42
Core PCE	-64.60	-71.58	1	0

Notes: According to the prior distributions, the probability of determinacy is 0.527.

Yet, the upshot differs depending on which measure of inflation we employ in the estimation. Take Bernanke's (2015) suggestion that Taylor's counterfactual experiment should have been performed with core PCE. When making this choice, the posterior probability for our sample concentrates all of its mass in the determinacy region. This result flags that the Federal Reserve had not been responding passively to inflation during this period.

However, the Humphrey-Hawkins reports to Congress document that the Federal Reserve based monetary policy deliberations on headline PCE from the beginning of 2000 until mid-2004. Since Taylor is particularly critical of the monetary policy from 2002 to 2004, we next measure inflation using headline PCE data. We repeat the estimation and the finding is now ambiguous: the probability of determinacy is 0.58. Phrased alternatively, we cannot dismiss the possibility of indeterminacy.

Table 1 also reports the posterior estimates of the model parameters of the respectively favored models for CPI and for core PCE inflation.[6] The estimated policy rule's response to inflation, ψ_π, which essentially governs the indeterminacy, differs significantly depending on the way we measure inflation. In particular, when basing the estimation on CPI, the posterior mean equals 0.84 (with 90-percent interval $[0.61, 0.98]$). This result indicates that monetary policy violated the Taylor principle over the 2002-2007 period or in the words of Taylor:

[6] The appendix reports results for parameter estimates when using headline PCE inflation data.

> "[t]he responsiveness appears to be at least as low as in the late 1960s and 1970s." [Taylor, 2007, 469]

The opposite result ensues when using core PCE. In that case, the posterior mean of ψ_π is well above one at 3.01 (with 90-percent interval $[1.97, 4.17]$). In sum, we find that indeterminacy outcomes are dependent on the measure of inflation that is used. In fact, this lines up with the Taylor and Bernanke debate.

4 Sensitivity analysis

We now investigate the sensitivity of our results in various directions. The robustness checks involve (i) testing for indeterminacy on rolling windows, (ii) estimating the policy parameters only, (iii) alternative priors for ψ_π, (iv) alternative measure of output and inflation, and (v) serially correlated monetary policy shocks.

Rolling windows The size of our sample is undeniably short. So first and foremost, we want to assess the extent to which our results might be an artifact of the small sample. To do so, we re-estimate the model on rolling windows starting in the mid-1960's, and keeping the size of the windows fixed at 23 quarters to match the number of observations in our period of interest. Thus the first window is 1966:I-1971:III. We move the window forward one quarter at a time, and re-estimate all parameters each time.[7] Here we just consider CPI inflation as the Federal Reserve only began to base its monetary policy deliberations on PCE and core PCE in the 2000s. Moreover, doing so makes our results directly comparable to Lubik and Schorfheide (2004). Figure 1 presents the evolution of the posterior probability of determinacy for the U.S. economy from 1966:I to 2008:III. The end point is chosen to avoid obvious complications that emanate from hitting the zero lower bound. The graph suggests that the U.S. economy was likely in a state of indeterminacy during the 1970s. Thereafter, beginning with the Volcker disinflation policies, the economy shifted back to a determinate equilibrium. These findings are consistent with related studies such as Clarida, Gali

[7] This approach to estimate linear DSGE models was recently promoted by Canova (2009), Canova and Ferroni (2011a) and Castelnuovo (2012a,b). Rolling window estimation provides two benefits. It allows us to uncover time-varying patterns of the model's parameters, in particular, of the monetary policy coefficients. At the same time, the procedure permits us to remain within the realm of linear models and apply standard Bayesian methods.

and Gertler (2000), Lubik and Schorfheide (2004) and Coibion and Gorodnichenko (2011).[8] We take this correspondence as a justification for estimating our model on a short window.[9] Our paper documents a second shift after the 2001 slump now from determinacy to indeterminacy.

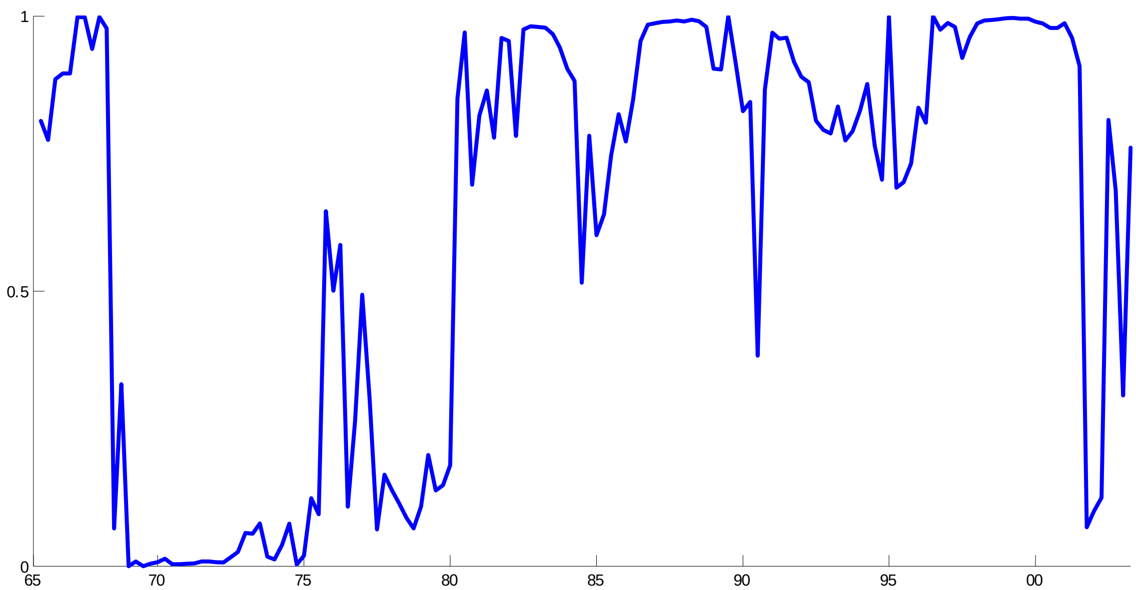

Figure 1: Probability of determinacy using rolling window estimation. The figure plots the probability at the first quarter of a window.

Estimating the policy parameters only As a further robustness check to address the small sample issue, we only estimate the policy parameters over the 2002-2007 period. More concretely, we exclusively estimate the three Taylor rule parameters along with the standard deviation of the monetary policy shock (as well as the sunspots related parameters, i.e. the **M**s and σ_ζ, for the indeterminacy version of the model). As for the other parameters, all were calibrated at the posterior means obtained from estimating the determinate model over the period 1991:II to 2001:IV.

[8] Figure 1 is comparable to Coibion and Gorodnichenko (2011, Figure 4). They report a moving average of the determinacy probability which makes their series smoother than ours. Coibion and Gorodnichenko (2011) use a model with trend inflation. We will explore such model in section 6.

[9] We furthermore experimented with the window length and the results appear to be robust.

The reason for beginning right after the 1990-91 recession is closely connected to Figure 1: it comfortably rules out indeterminacy even for "short" periods. Table 3 reports strong evidence for indeterminacy not only when we measure inflation with CPI but also with PCE. However, as before, the posterior probability puts all its weight on determinacy when inflation is measured using Core PCE.

Table 3. Determinacy versus Indeterminacy (Robustness)

		Log-data density		Probability	
Inflation measure		Det.	Indet.	Det.	Indet.
CPI	Policy parameters only	-99.97	-95.50	0.01	0.99
	Alternative prior for ψ_π	-95.04	-93.58	0.19	0.81
	CBO output gap	-97.89	-95.85	0.12	0.88
	Output growth	-93.29	-89.58	0.02	0.98
	AR(1) policy shocks	-89.51	-85.68	0.02	0.98
PCE	Policy parameters only	-99.36	-88.79	0.07	0.93
	Alternative prior for ψ_π	-85.04	-85.98	0.72	0.28
	CBO output gap	-88.08	-88.18	0.53	0.47
	Output growth	-82.89	-81.80	0.25	0.75
	AR(1) policy shocks	-77.59	-77.25	0.42	0.58
Core PCE	Policy parameters only	-63.49	-69.49	1	0
	Alternative prior for ψ_π	-64.47	-71.74	1	0
	CBO output gap	-68.53	-73.63	0.99	0.01
	Output growth	-62.54	-67.58	1	0
	AR(1) policy shocks	-53.91	-62.09	1	0

Alternative priors One possible drawback to using a small sample size is that the prior might *speak louder* than the data. To make our empirical analysis transparent, the priors we employ in our baseline estimation (Table 1) were set identical to the ones used by Lubik and Schorfheide (2004). Accordingly, our baseline specification implies a prior probability of determinacy equal to 0.53. To assess the sensitivity of our results to the priors, we alter the prior distribution for the key parameter that drives indeterminacy. Specifically, we change the prior mean of ψ_π from 1.1 to 1.3 and in doing so we ramp up the prior probability of determinacy from 0.53 to 0.7. Thus, the indeterminacy test will now find it harder to favor indeterminacy. Table 3 reports the posterior probabilities of (in-)determinacy under this alternative prior for each measure of inflation. The results remain largely unaltered. For example, the odds of indeterminacy versus determinacy are still five to one when estimating the model using CPI inflation. This finding provides some further support for our results.

Alternative measures of output To make our baseline analysis comparable with Lubik and Schorfheide (2004), we used HP-filtered GDP to measure output fluctuations. However, as argued by Canova (1998), Gorodnichenko and Ng (1998) among others, filtered data may induce spurious results. Accordingly, we now consider two alternative ways to gauge real economic activity. First, we replace HP-trend output by the Congressional Budget Office's estimate of potential output as in Belongia and Ireland (2015) and others. Table 3 suggests that, again, our results remain robust. Second, we use output growth instead of an output gap measure. To this end, we assume that the artificial economy now features non-stationary technology – it follows a deterministic trend as in Mattesini and Nisticò (2010) or Ascari, Castelnuovo and Rossi (2011).[10] Also, we no longer estimate the intertemporal rate of substitution, $1/\tau$, and instead set it equal to one to make the model consistent with balanced growth. Then, Table 3 shows that when using output growth, the case for indeterminacy becomes even stronger for CPI and PCE, yet, it remains unchanged when measuring inflation via core PCE data.[11]

Serially correlated monetary policy shocks Our findings so far have lend some support to the conjecture that monetary policy was extra easy following the 2001 recession. Our exercise interprets this view as a reduction in the Federal Reserve's systematic response to the inflation gap (thereby leading to indeterminacy of the rational expectations equilibrium). However, alternatively, extended periods of too low interest rates could also arise due to discretionary deviations from the monetary policy rule (see also Rudebusch, 2002, Groshenny, 2013, and Belognia and Ireland, 2016). To assess the robustness of our interpretation, we next allow the monetary policy shocks to be serially correlated. Specifically, we assume that the policy shocks

[10] The measurement equation now writes $\gamma_{yt}^{obs} = \gamma + \Delta\widehat{y}_t$ where γ_{yt}^{obs} is the observed growth rate of output, γ stands for the steady state growth rate and $\Delta\widehat{y}_t$ is the first-differenced logarithm of stationarized model output. The prior distribution of γ is $\mathsf{N}(0.5, 0.1)$.

[11] Given the indicated issues of HP-filtered data and the essentially unchanged results when employing output growth, the remainder of this paper will concentrate on output growth.

follow the AR(1) process

$$\epsilon_{R,t} = \rho_{\epsilon_R}\epsilon_{R,t-1} + v_t \qquad 0 \leq \rho_{\epsilon_R} < 1$$

where v_t is $i.i.d.\mathsf{N}(0, \sigma_v^2)$ and jointly estimate the autocorrelation parameter, ρ_{ϵ_R}, and the standard deviation of the shock, σ_v^2, along with the other parameters of the model.[12] Table 3 confirms that our results remain unaltered: we still cannot rule out passive responsiveness to inflation and thereby the possibility of indeterminacy.

GDP Deflator While not mentioned in the Humphrey-Hawkins reports to have informed Federal Reserve's policy deliberations during the 2000s, we lastly re-do the analysis with the GDP deflator as the inflation measure (as in Smets and Wouters, 2007). Then, the log-data densities are very close at -73.26 for determinacy and -74.16 for indeterminacy. Phrased differently, the posterior probabilities of determinacy and indeterminacy are 71% versus 29% and again we cannot rule out indeterminacy.

5 Which measure of inflation to choose?

Our baseline estimations have delivered mixed evidence regarding the probability of indeterminacy for the 2002:I to 2007:III period. The results are consistently dependent on the specific inflation measure used in the estimation – only with core PCE series can we comfortably rule out indeterminacy. However, each inflation proxy may only provide an imperfect indicator of the model concept. Put differently, all three measures of inflation may contain relevant information. In this line of thinking, we will now depart from the assumption that model inflation is measured by a single series and draw on Boivin and Giannoni's (2006) *data-rich environment* application of dynamic factor analysis to DSGE models.[13] In a nutshell, we want to exploit the information from all the inflation series in the estimation to deliver more robust results. We treat the model concept of inflation as the unobservable common factor for which data series are imperfect proxies.

[12]The AR(1) coefficient of the policy shock follows a beta prior with mean 0.5 and standard deviation 0.2.

[13]Canova and Ferroni (2011b) and Castelnuovo (2013) are recent applications.

More concretely, the estimation involves the transition equation (4)

$$s_t = \Phi(\theta)s_{t-1} + \Phi_\varepsilon(\theta, \widetilde{M})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t$$

or its determinacy equivalent

$$s_t = \Phi^D(\theta)s_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$$

and the measurement equation

$$\begin{bmatrix} \Delta GDP_t \\ FFR_t \\ \mathbf{X}_t \end{bmatrix} = \begin{bmatrix} \gamma \\ r+\pi \\ \underset{4\times 1}{\mathbf{0}} \end{bmatrix} + \begin{bmatrix} \mathbf{I}_2 & \underset{2\times 4}{\mathbf{0}} \\ \underset{4\times 2}{\mathbf{0}} & \mathbf{\Lambda} \end{bmatrix} \begin{bmatrix} \Delta y_t \\ 4R_t \\ \boldsymbol{\pi}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \mathbf{u}_t \end{bmatrix}. \tag{6}$$

Here ΔGDP_t stands for the growth rate of per-capita real GDP, FFR_t denotes the Federal Funds rate, $\mathbf{X}_t \equiv [\Delta CPI_t, \Delta PCE_t, \Delta corePCE_t, \Delta DEF_t]'$ is the vector of empirical inflation proxies[14], $\mathbf{\Lambda}$ =diag$(\lambda_{CPI}, \lambda_{PCE}, \lambda_{corePCE}, \lambda_{DEF})$ is a 4×4 matrix of factor loadings relating the latent model concept of inflation to the four indicators, $\boldsymbol{\pi}_t \equiv 4[\pi_t, \pi_t, \pi_t, \pi_t]'$ and $\mathbf{u}_t = [u_t^{CPI}, u_t^{PCE}, u_t^{corePCE}, u_t^{DEF}]' \sim i.i.d.(\mathbf{0}, \mathbf{\Sigma})$ is a vector of serially and mutually uncorrelated indicator-specific measurement errors, with $\mathbf{\Sigma}$ =diag$(\sigma^2_{CPI}, \sigma^2_{PCE}, \sigma^2_{corePCE}, \sigma^2_{DEF})$.

We jointly estimate the parameters $(\mathbf{\Lambda}, \mathbf{\Sigma})$ of the measurement equation (6) along with the structural parameters θ. We calibrate π equal to 2.5 percent - a value roughly in line with the average of the sample means of the inflation series. We standardize the four indicators to have mean zero and unit variance. This standardization permits us to interpret the factor loadings, $\lambda_j s$, as correlations between the latent theoretical concept of inflation and the respective observables.[15] Our prior distribution for the loadings and measurement errors are $\lambda_j \sim Beta(0.50, 0.25)$ and $u_t^j \sim \mathsf{N}(0.10, 0.20)$ respectively. By employing a beta distribution, the support of the λ_j is restricted to the open interval $(0, 1)$ which is a necessary sign restriction.

Table 4 reports the resulting log-data densities which are -162.50 for determinacy and -161.83 for indeterminacy. Phrased differently, the posterior probabilities of determinacy and indeterminacy are 34% versus 66%, hence, we cannot rule out indeterminacy.[16]

[14] DEF is the acronym for the GDP Deflator.

[15] See Geweke and Zhou (1996) and Forni, Hallin, Lippi and Reichlin (2000).

[16] We also replicated Lubik and Schorfheide (2004) with the DSGE factor model approach. The outcomes of the indeterminacy test for the pre-Volcker and post-1982 sample periods remain unaltered to this extension.

Table 4: Determinacy versus Indeterminacy (DSGE-Factor)

Log-data density		Probability	
Determinacy	Indeterminacy	Determinacy	Indeterminacy
-162.50	-161.83	0.34	0.66

Notes: The prior predictive probability of determinacy is 0.527.

Table 5 - Parameter Estimation Results (DSGE-Factor)

	Determinacy		Indeterminacy	
	Mean	[5th pct, 95thpct]	Mean	[5th pct, 95th pct]
ψ_π	2.13	[1.29,3.13]	0.80	[0.61,0.98]
ψ_y	0.30	[0.07,0.65]	0.21	[0.05,0.45]
ρ_R	0.81	[0.72,0.88]	0.81	[0.73,0.88]
r^*	1.00	[0.45,1.67]	1.23	[0.57,2.00]
κ	0.74	[0.41,1.15]	1.00	[0.57,1.49]
γ	0.53	[0.45,0.62]	0.51	[0.44,0.58]
ρ_g	0.79	[0.68,0.87]	0.60	[0.45,0.74]
ρ_z	0.68	[0.50,0.85]	0.70	[0.54,0.84]
ρ_{gz}	0.14	[-0.33,0.70]	-0.31	[-0.74,0.15]
$M_{R\zeta}$			-0.31	[-1.53,1.17]
$M_{g\zeta}$			-1.77	[-2.59,-0.95]
$M_{z\zeta}$			0.30	[0.01,0.62]
σ_R	0.18	[0.13,0.25]	0.16	[0.12,0.21]
σ_g	0.19	[0.14,0.27]	0.28	[0.18,0.42]
σ_z	0.69	[0.50,0.94]	0.73	[0.53,1.00]
σ_ζ			0.18	[0.12,0.27]
λ_{CPI}	0.76	[0.55,0.93]	0.57	[0.37,0.79]
λ_{PCE}	0.79	[0.59,0.95]	0.59	[0.40,0.82]
$\lambda_{CorePCE}$	0.28	[0.07,0.52]	0.21	[0.06,0.40]
λ_{DEF}	0.53	[0.31,0.77]	0.41	[0.23,0.64]
σ_{CPI}	0.31	[0.20,0.43]	0.32	[0.22,0.43]
σ_{PCE}	0.18	[0.10,0.31]	0.18	[0.10,0.29]
$\sigma_{CorePCE}$	0.91	[0.72,1.14]	0.91	[0.72,1.14]
σ_{DEF}	0.71	[0.56,0.90]	0.70	[0.56,0.88]

Notes: The table reports posterior means and 90 percent probability intervals of the DSGE-Factor model parameters.

Table 5 reports the posterior estimates of the model parameters along with the factor loadings (i.e. the correlations between the latent factor and the proxies) as well as the standard deviations of the measurement errors. Conditional on both determinacy and indeterminacy the loadings on CPI and PCE are about three times as large as the loading on core PCE. Furthermore, there is evidence of substantial indicator-specific component for core PCE as evident in the high standard deviation of its measurement error. These results imply that CPI and PCE provide better indicators of the latent concept of inflation, while core PCE, despite being promoted by Bernanke (2015), is less informative. In other words, while core PCE might better fit the Federal Reserve's behavior in isolation, the other inflation measures are more consistent with the New Keynesian model as a whole.

6 Trend inflation

So far, our analysis has assumed that the U.S. economy was reasonably approximated by the standard New Keynesian model linearized around a zero inflation steady state. However, the Federal Reserve's implicit inflation target as well as the average inflation rate during the Great Moderation period was around two to three percent (depending on the chosen price index). Thus, we extend the baseline model to allow for positive trend inflation.

This extension becomes meaningful for at least two further reasons. Firstly, Ascari and Ropele (2009) show how positive trend inflation alters the determinacy properties of the model: by increasing the cost of price dispersion, higher trend inflation induces a negative relationship between steady state output and inflation. This long-run Phillips curve affects the Taylor principle in a way that shrinks the determinacy region. Moreover, Ascari and Sbordone (2014) show that this effect also depends on the degree of price stickiness. Since estimates of trend inflation and price stickiness will both depend on the particular measures of inflation being used, the extension for trend inflation makes the determinacy tests even more relevant. Secondly, it is well known that the determinate plain-vanilla New Keynesian model features a poor internal propagation mechanism while it potentially exhibits richer dynamics under indeterminacy. Accordingly, the posterior mass might be biased toward the indeterminacy region (see Beyer and Farmer, 2007). Trend inflation generates more endogenous persistence of inflation and output even in the determinacy case and this further justifies our extension.

The estimation is based on a version of Ascari and Sbordone's (2014) Generalized New Keynesian model (GNK). Unlike Ascari and Sbordone, we assume deterministic growth and we replace their labor supply disturbance by a discount factor shock, d_t, as our stand-in for demand shocks. Also, our Taylor rule involves responses to the output gap instead of log-deviations from the steady state. This then makes our setup similar to Hirose, Kurozumi and Van Zandweghe (2015).[17] The log-linearized (detrended) model consists of the Euler equation

$$y_t = E_t y_{t+1} - (R_t - E_t \pi_{t+1}) + d_t - d_{t+1}$$

where we have set the intertemporal rate of substitution equal to one to make the model compatible with balanced growth as well as the Taylor rule

$$R_t = \rho_R R_{t-1} + (1-\rho_R)(\psi_\pi \pi_t + \psi_y [y_t - z_t]) + \epsilon_{R,t} \qquad 0 \leq \rho_R < 1$$

to capture the central bank's behavior. The supply side is no longer summarized by a single Phillips curve expression but rather it consists of the following three equations for inflation, an auxiliary variable, ψ_t, and price dispersion, s_t:

$$\begin{aligned}
\pi_t &= \varkappa E_t \pi_{t+1} + \vartheta \left[\varphi s_t + (1+\varphi) y_t - (1+\varphi) z_t\right] - \varpi E_t \psi_{t+1} + \varpi d_t \\
\psi_t &= (1-\xi\beta\pi^\varepsilon)\left[\varphi s_t + (1+\varphi)(y_t - z_t) + d_t\right] + \xi\beta\pi^\varepsilon \left[E_t \psi_{t+1} + \varepsilon E_t \pi_{t+1}\right] \\
s_t &= \varepsilon \xi \pi^\varepsilon \left(1 - \frac{1-\xi\pi^\varepsilon}{\pi - \xi\beta\pi^\varepsilon}\right) \pi_t + \xi \pi^\varepsilon s_{t-1}
\end{aligned}$$

where $\vartheta \equiv (1-\xi\pi^{\varepsilon-1})(1-\xi\beta\pi^\varepsilon)/\xi\pi^{\varepsilon-1}$, $\varkappa \equiv \beta\left[1 + \varepsilon(\pi - 1)(1-\xi\pi^{\varepsilon-1})\right]$, and $\varpi \equiv \beta(1-\pi)(1-\xi\pi^{\varepsilon-1})$. The term ξ denotes the Calvo-parameter and β stands in for the steady state discount factor. We set the Frisch elasticity of labor supply, φ, equal to one and calibrate the elasticity of substitution $\varepsilon = 11$ such that the steady state mark-up equals ten percent.

As mentioned above, the GNK model exhibits richer dynamics and the usual Taylor principle ($\psi_\pi > 1$) is no longer a sufficient condition for local determinacy of equilibrium. Due to the higher-order dynamics of the GNK model and our assumption

[17] They, however, assume firm-specific labor as well as stochastic growth.

of a unit Frisch elasticity of labor supply, it is not possible to analytically derive the indeterminacy conditions. To continue solving the model via Lubik and Schorfheide's (2004) continuity solution (where $\mathbf{M}^*(\theta)$ is selected that the responses of the endogenous variables to the fundamental shocks are continuous at the boundary between the determinacy and indeterminacy region) one needs to resort to numerical methods. In particular, we follow Hirose's (2014) numerical solution strategy for finding the boundary between determinacy and indeterminacy by perturbing the parameter ψ_π in the monetary policy rule.

As before we use the growth rate of GDP, the Federal Funds rate and the three measures of inflation sequentially. Table 6 reports the priors used for the estimation while Table 7 provides the marginal data densities along with the posterior model probabilities. The emerging results parallel our earlier findings. When basing the estimation on CPI, the U.S. economy was very likely in an indeterminacy region, however, the opposite holds, again, under core PCE. Notably, as mentioned above, the posterior estimate of trend inflation under CPI is higher than under core PCE while the Calvo parameter is smaller implying more flexible prices under CPI.

Lastly, we investigate the sensitivity of our results to Coibion and Gorodnichenko's (2011) Taylor rule that allows for interest rate smoothing of order two, as well as a response to inflation, output growth, and the output gap. Coibion and Gorodnichenko document a shift in the Federal Reserve's response from output gap to output growth for the Great Moderation period and also show that the two lags of interest rate are required to remove the serial correlation in the monetary policy shocks. Thus, we re-estimate the GNK model by replacing the standard policy rule with the following formulation:

$$R_t = \rho_{R_1} R_{t-1} + \rho_{R_2} R_{t-2} + (1 - \rho_{R_1} - \rho_{R_2})(\psi_\pi \pi_t + \psi_y [y_t - z_t] + \psi_{gy} \Delta y_t) + \epsilon_{R,t}.$$

Even though the posterior probabilities of indeterminacy are now lower across the board, Table 7 shows that the only case in which we can confidently rule out the possibility of indeterminacy is when we use core PCE. Apart from the parameter estimates of the responsiveness to output growth, ψ_{gy}, and the interest rate lags, ρ_{R1} and ρ_{R2}, all other parameter estimates remain essentially changed.

Table 6 - Priors and posteriors for DSGE parameters.

				Posterior Mean [5th pct, 95th pct]			
				Standard TR		Alternative TR	
Name	Range	Density	Prior Mean (Std. Dev.)	CPI Ind.	CorePCE Det.	CPI Ind.	CorePCE Det.
ψ_π	$\mathbb{R}^+$	Gamma	1.40 (0.50)	0.93 [0.82,1.00]	2.65 [1.66,3.77]	0.94 [0.84,1.00]	2.53 [1.64,3.55]
ψ_y	$\mathbb{R}^+$	Gamma	0.25 (0.15)	0.25 [0.07,0.54]	0.35 [0.09,0.72]	0.25 [0.07,0.52]	0.30 [0.07,0.65]
ψ_{gy}	$\mathbb{R}^+$	Gamma	0.25 (0.15)			0.33 [0.11,0.59]	0.35 [0.10,0.70]
ρ_R	[0,1)	Beta	0.50 (0.20)	0.73 [0.63,0.82]	0.76 [0.64,0.85]		
ρ_{R1}	$\mathbb{R}$	Normal	1.00 (0.20)			1.09 [0.85,1.32]	1.13 [0.88,1.36]
ρ_{R2}	$\mathbb{R}$	Normal	0.00 (0.20)			−0.35 [−0.55,−0.14]	−0.35 [−0.57,−0.12]
π	$\mathbb{R}^+$	Gamma	2.50 (1.00)	2.21 [1.03,3.73]	1.89 [1.52,2.26]	2.12 [1.02,3.49]	1.93 [1.54,2.35]
r	$\mathbb{R}^+$	Gamma	2.00 (1.00)	1.01 [0.45,1.71]	1.27 [0.69,1.91]	0.93 [0.40,1.59]	1.22 [0.63,1.87]
γ	$\mathbb{R}$	Normal	0.50 (0.10)	0.49 [0.44,0.53]	0.55 [0.48,0.62]	0.49 [0.45,0.54]	0.55 [0.48,0.62]
ξ	[0,1)	Beta	0.70 (0.10)	0.26 [0.19,0.34]	0.64 [0.51,0.74]	0.31 [0.23,0.40]	0.65 [0.53,0.74]
ρ_d	[0,1)	Beta	0.70 (0.10)	0.71 [0.54,0.85]	0.82 [0.72,0.90]	0.73 [0.56,0.87]	0.82 [0.73,0.90]
ρ_z	[0,1)	Beta	0.70 (0.10)	0.70 [0.58,0.82]	0.76 [0.61,0.89]	0.69 [0.55,0.81]	0.75 [0.59,0.88]
$M_{R\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	−0.74 [−1.82,0.42]		−0.84 [−1.97,0.39]	
$M_{d\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	−1.47 [−2.57,0.13]		−1.15 [−2.62,0.61]	
$M_{z\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	1.98 [1.36,2.65]		1.68 [1.01,2.39]	
σ_R	$\mathbb{R}^+$	IG	0.50 (∞)	0.19 [0.13,0.26]	0.15 [0.11,0.21]	0.18 [0.12,0.26]	0.14 [0.10,0.20]
σ_d	$\mathbb{R}^+$	IG	0.50 (∞)	0.36 [0.15,0.82]	0.74 [0.51,1.09]	0.25 [0.13,0.47]	0.70 [0.46,1.04]
σ_z	$\mathbb{R}^+$	IG	0.50 (∞)	0.46 [0.35,0.59]	0.56 [0.37,0.85]	0.48 [0.36,0.63]	0.58 [0.48,0.62]
σ_ζ	$\mathbb{R}^+$	IG	0.50 (∞)	0.27 [0.15,0.47]		0.31 [0.16,0.56]	

Notes: The inverse gamma priors are of the form $p(\sigma|\upsilon,s) \propto \sigma^{-\upsilon-1}e^{-\frac{\upsilon s^2}{2\sigma^2}}$, where $\nu = 2$ and $s = 0.282$.

Table 7: Determinacy versus Indeterminacy (Trend Inflation)

		Log-data density		Probability	
Inflation measure		Det.	Indet.	Det.	Indet.
CPI	Standard Taylor rule	-91.38	-87.13	0.02	0.98
	Alternative Taylor rule	-85.16	-83.25	0.13	0.87
PCE	Standard Taylor rule	-81.54	-82.01	0.62	0.38
	Alternative Taylor rule	-75.79	-77.41	0.83	0.17
Core PCE	Standard Taylor rule	-61.13	-64.53	0.97	0.03
	Alternative Taylor rule	-56.68	-60.75	0.98	0.02

Notes: The prior predictive probability is 0.539 for the standard rule and 0.503 for the alternative rule.

7 Concluding remarks

Using the Taylor rule as a benchmark for evaluating the Federal Reserve's interest-rate setting decisions, some commentators have argued that monetary policy was too accommodative during the 2002-2005 period. Along these lines, this paper estimates a New Keynesian model of the U.S. economy for the time following the 2001 slump. Our assessment of the Federal Reserve's performance varies with the measure of inflation that is put into the model estimation. When measuring inflation with CPI or PCE, we find some support for the view that monetary policy during these years was extra easy and led to equilibrium indeterminacy. Instead, if the estimation involves core PCE, monetary policy comes out as active and the evidence for indeterminacy dissipates. This result remains very robust to several extensions of the model including trend inflation. Our take on these diverging results is that each inflation series only provides an imperfect proxy for the model's concept of inflation. We re-formulate the artificial economy as a factor model where the theory's concept of inflation is the common factor to the alternative empirical inflation series. Again, extra easy monetary policy as well as indeterminacy cannot be ruled out. In sum, while not completely resolving the ongoing debate between Bernanke, Taylor and others, our study sheds further light on the effects of U.S. monetary policy during the years leading up to the Great Recession.

We choose to make these arguments while staying in a relatively standard model.

This choice enables to establish a bridge from existing research to our study which we believe is important given the short data sample that we consider. We specifically do not introduce asset markets into the model or in the estimation part – potentially tilting our estimates towards determinacy. Thus, in terms of possible extensions, it would be worthwhile to introduce asset markets – ideally for housing – into the model and in the econometric analysis. We plan to pursue this research in the near future.

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Appendix for "Monetary Policy and Indeterminacy after the 2001 Slump" (not for publication)*

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1 Roadmap

This Appendix presents several extensions and results of robustness checks to our paper. Section 2 describes the models used in our analysis. Section 3 explains the solution method under indeterminacy. Section 4 depicts our estimation strategy. Finally, section 5 provides some additional results.

2 Framework of the structural analysis

In this section we outline the two main models used in the paper. We begin with the plain-vanilla version of the New Keynesioan model.

2.1 Baseline New Keynesian Model

The artificial economy can be summarized in terms of the familiar linearized three equations of the plain-vanilla New Keynesian (NK) model:

$$y_t = E_t y_{t+1} - \tau(R_t - E_t \pi_{t+1}) + g_t \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(y_t - z_t) \tag{2}$$

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)(\psi_\pi \pi_t + \psi_y [y_t - z_t]) + \varepsilon_{R,t.} \tag{3}$$

Here y_t stands for the output, R_t denotes the interest rate and π_t symbolizes the inflation rate. E_t represents the expectations operator. Equation (1) is the dynamic IS-relation reflecting an Euler equation in which τ can be interpreted as the intertemporal elasticity of substitution. Equation (2) describes the expectational Phillips

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curve where $0 < \beta < 1$ is the agents' discount factor. Finally, equation (3) describes monetary policy, i.e. a Taylor-type nominal interest rate rule in which $\psi_\pi$and ψ_y are chosen by the central bank and echo its responsiveness to inflation and the output gap, $y_t - z_t$. $0 < \rho_R < 1$ is the usual smoothing term. $\epsilon_{R,t}$ denotes an exogenous monetary policy shock whose standard deviation is given by σ_R. Fundamental disturbances involve exogenous shifts of the Euler equation captured by the process g_t and shifts of the marginal costs of production captured by z_t. Both variables follow AR(1) processes:

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t} \qquad 0 < \rho_g < 1 \tag{4}$$

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t} \qquad 0 < \rho_z < 1. \tag{5}$$

The standard deviations for the demand and supply shocks are denoted by σ_g and σ_z. We allow for a non-zero correlation, $\rho_{g,z}$, between the demand and supply innovations.

Indeterminacy implies that fluctuations in economic activity can be driven by arbitrary, self-fulfilling changes in people's expectations (i.e. sunspots). Concretely, in the above New Keynesian model this can occur if the central bank only irresolutely responds to inflation changes. The precise analytical condition for indeterminacy corresponds to $\phi_\pi < 1 - \phi_y (1 - \beta) / \kappa$.

2.2 Generalized New Keynesian Model

The estimation is based on a version of Ascari and Sbordone's (2014) Generalized New Keynesian model (GNK). Unlike Ascari and Sbordone, we assume deterministic growth and we replace their labor supply shock by disturbances of the discount factor, d_t, as our stand-in for demand shock. Also, our Taylor rule involves responses to the output gap instead of log-deviations from the steady state. This then makes our setup similar to Hirose, Kurozumi and Van Zandweghe (2015).[1] The log-linearized model consists of the Euler equation

$$y_t = E_t y_{t+1} - (R_t - E_t \pi_{t+1}) + d_t - d_{t+1}$$

where we have set the intertemporal rate of substitution equal to one to make the model compatible with balanced growth as well as the Taylor rule

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)(\psi_\pi \pi_t + \psi_y [y_t - z_t]) + \epsilon_{R,t} \qquad 0 \leq \rho_R < 1$$

to capture the central bank's behavior. The supply side is no longer summarized by a single Phillips curve expression but rather it consists of the following three equations for inflation, an auxillary variable, ψ_t, and price dispersion, s_t:

$$\begin{aligned}
\pi_t &= \varkappa E_t \pi_{t+1} + \vartheta \left[\varphi s_t + (1 + \varphi) y_t - (1 + \varphi) z_t \right] - \varpi E_t \psi_{t+1} + \varpi d_t \\
\psi_t &= (1 - \xi \beta \pi^\varepsilon) \left[\varphi s_t + (1 + \varphi)(y_t - z_t) + d_t \right] + \xi \beta \pi^\varepsilon \left[E_t \psi_{t+1} + \varepsilon E_t \pi_{t+1} \right] \\
s_t &= \varepsilon \xi \pi^\varepsilon \left[1 - \frac{(1 - \xi \pi^\varepsilon)}{(1 - \xi \beta \pi^{\varepsilon - 1}) \pi} \right] \pi_t + \xi \pi^\varepsilon s_{t-1}
\end{aligned}$$

[1] They, however, assume firm-specific labor as well as stochastic growth.

where $\vartheta \equiv (1-\xi\pi^{\varepsilon-1})(1-\xi\beta\pi^{\varepsilon})/\xi\pi^{\varepsilon-1}$, $\varkappa \equiv \beta\left[1+\varepsilon(\pi-1)(1-\xi\pi^{\varepsilon-1})\right]$, and $\varpi \equiv \beta(1-\pi)(1-\xi\pi^{\varepsilon-1})$. The term ξ denotes the Calvo-parameter and β stands in for the steady state discount factor. We set the Frisch elasticity of labor supply, φ, equal to one and calibrate the elasticity of substitution $\varepsilon = 11$ such that the steady state mark-up equals ten percent.

3 Rational-expectations solution under indeterminacy

Heree we will outline the solution to this model which follows Lubik and Schorfheide (2003). Let us denote by η_t the vector of one-step ahead expectational errors. Moreover, define s_t as the vector of endogenous variables and ε_t as vector of fundamental shocks. Then, the linear rational expectation system can be compactly written as

$$\Gamma_0(\theta)s_t = \Gamma_1(\theta)s_{t-1} + \Psi(\theta)\varepsilon_t + \Pi(\theta)\eta_t \tag{6}$$

where $\Gamma_0(\theta)$, $\Gamma_1(\theta)$, $\Psi(\theta)$, and $\Pi(\theta)$ are appropriately defined coefficient matrices. We follow Sims' (2002) solution algorithm that was revisited by Lubik and Schorfheide (2003). This has the advantage of being general and explicit in dealing with expectation errors since it makes the solution suitable for solving and estimating models which feature multiple equilibria. In particular, under indeterminacy η_t will be a linear function of the fundamental shocks and the purely extrinsic sunspot disturbances, ζ_t. Hence, the full set of solutions to the LRE model entails

$$s_t = \Phi(\theta)s_{t-1} + \Phi_\varepsilon(\theta, \widetilde{M})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t \tag{7}$$

where $\Phi(\theta)$, $\Phi_\varepsilon(\theta, \widetilde{M})$ and $\Phi_\zeta(\theta)^2$ are the coefficient matrices.[3] The sunspot shock satisfies $\zeta_t \sim i.i.d.\mathsf{N}(0, \sigma_\zeta^2)$. Accordingly, indeterminacy can manifest itself in one of two different ways: (i) pure extrinsic non-fundamental disturbances can affect model dynamics through endogenous expectation errors and (ii) the propagation of fundamental shocks cannot be uniquely pinned down and the multiplicity of equilibria affecting this propagation mechanism is captured by the arbitrary matrix $\widetilde{M}$.

Following Lubik and Schorfheide (2004) we replace $\widetilde{M}$ with $M^*(\theta)+M$ and in the subsequent empirical analysis set the prior mean for M equal to zero. The particular solution employed in their paper selects $M^*(\theta)$ by using a least squares criterion to minimize the behaviour of the model under determinacy and indeterminacy by assuming that it remains unchanged across the boundary. "Behaviour" needs be described in some meaningful way and we follow them by choosing $M^*(\theta)$ such that the response of the endogenous variables to fundamental shocks, $\partial s_t/\partial\varepsilon_t'$, are continuous at the boundary between the determinacy and the indeterminacy region. While for the small-scale NK model we have an analytical solution for the boundary, however, we resort to a numerical procedure for the GNK model to find the boundary by perturbing the parameter ψ_π in the monetary policy rule.

[2] Lubik and Schorfheide (2003) express this term as $\Phi_\zeta(\theta, M_\zeta)$, where M_ζ is an arbitrary matrix. For identification purpose, they impose the normalization such that $M_\zeta = I$.

[3] Under determinacy, the solution boils down to $s_t = \Phi^D(\theta)s_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$.

4 Estimation Strategy

We employ Bayesian techniques for estimating the parameters of the model and test for indeterminacy using posterior model probabilities. In order to construct a likelihood function the DSGE model is turned into a Bayesian model. Toward that purpose we need to define a set of measurement equations that relate the elements of s_t to a set of observables x_t which is given by

$$x_t = \begin{bmatrix} \gamma \\ \pi^* \\ r^* + \pi^* \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 \end{bmatrix} s_t \tag{8}$$

where π^*, $r^*$and γ^* are annualized steady state inflation, annualized steady state real interest rates and quarterly steady state growth rate of real GDP per capita respectively.[4] Equations (7) and (8) provide a state-space representation for the linearized DSGE model that allows us to continue to apply standard Bayesian methodologies.

First priors are described by a density function of the form

$$p(\theta_S|S)$$

where $S \in \{D, I\}$ stands for a specific model, θ_S represents the parameter of the model S, $p(.)$ stands for probability density function. Next, the likelihood function describes the density of the observed data:

$$\mathcal{L}(\theta_S|X_T, S) \equiv p(X_T|\theta_S, S)$$

where X_T are the observations until period T. By using Bayes theorem we can combine the prior density and the likelihood function to get the posterior density:

$$p(\theta_S, X_T, S) = \frac{p(X_T|\theta_S, S)p(\theta_S|S)}{p(X_T, S)}$$

where $p(X_T|S)$ is the marginal marginal density of the data conditional on the model which is given by

$$p(X_T|S) = \int_{\theta_S} p(\theta_S; X_T)d\theta_S.$$

Finally, the posterior kernel corresponds to the numerator of the posterior density:

$$p(\theta_S|X_T, S) \propto p(X_T|\theta_S, S)p(\theta_S|S) \equiv \kappa(\theta_S|X_T, S).$$

We maximize the posterior kernel and find the posterior mode in the two regions of the parameter space using Sims' `csminwel`. The inverse Hessian is calculated at the posterior mode.[5] Next for each region of the parameter space we estimate the

[4] When using HP-filtered data to measure real activity γ^* is set to zero.

[5] For our rolling windown approach, if for a particular sample a region of the parameter space does not have a local mode, we use the inverse Hessian obtained from the nearest previous sample for that region.

likelihood function with the help of the Kalman filter and generate 250,000 draws with a random-walk Metropolis Hastings algorithm. The algorithm is tuned to achieve 25 to 30 percent acceptance rate. Half of the parameter draws are discarded to ensure convergence and the remaining draws are used to generate our results. The marginal data densities for the two regions are computed with Geweke's (1999) modified harmonic mean estimator.

5 Additional results

5.1 Baseline NK model with habit formation

It is well known that the determinate New Keynesian model features a poor internal propagation mechanism while the model potentially exhibits richer dynamics under indeterminacy. Accordingly, the posterior mass might be biased toward the indeterminacy region.[6] Hence, following Lubik and Schorfheide (2004), we extend the model by adding consumption habits. Log-data densities for the habit specification conditional on determinacy are reported in Table A1: the habit model fits better than the no-habit specification restricted to determinacy. The last column of Table A1 compares the respective posterior probabilities of the baseline model under indeterminacy and the habit model under determinacy. For example, when measuring inflation with CPI, the data favors the benchmark model under indeterminacy over the habit specification restricted to determinacy. Again, the results carry over from the benchmark exercise i.e. Table 2 in the paper.

Table A1: Benchmark Model versus Determinate Model with Habit

		Log-data density		
Inflation measure	Specification	Det.	Indet.	Probability
CPI	Benchmark	-95.48	-93.28	0.87
	Habit	-95.18		0.13
PCE	Benchmark	-85.42	-85.75	0.26
	Habit	-84.70		0.74
Core PCE	Benchmark	-64.60	-71.58	0
	Habit	-62.73		1

5.2 Estimation Results under PCE

According to the semi-annual monetary policy reports to Congress (Humphrey-Hawkins reports), the Federal Reserve has also been looking at headline PCE inflation from 2000 to 2004. Hence, we employ PCE to measure inflation while estimating our model and the evidence is mixed at best: the probability of determinacy is 0.58 and 0.62 for the small-scale NK model and the GNK model respectively. Phrased alternatively, we can neither exclude nor rule in indeterminacy.

[6] See the discussion between Beyer and Farmer (2007) and Lubik and Schorfheide (2007).

Table A2 reports posterior estimates of the small-scale NK model parameters under both determinacy and indeterminacy while Table A3 displays the estimates for the GNK model for both the standard Taylor rule and the alternative rule following Coibion and Gorodnichenko (2011). Most of our posterior estimates are in line with results reported in Table 3 and Table 12 respectively in the paper.

Table A2 - Parameter Estimation Results

	PCE (Indeterminacy)		PCE (Determinacy)	
	Mean	90-percent interval	Mean	90-percent interval
ψ_π	0.82	[0.58,0.97]	2.13	[1.30,3.09]
ψ_y	0.21	[0.05, 0.45]	0.27	[0.06,0.59]
ρ_R	0.83	[0.74, 0.90]	0.85	[0.77,0.91]
π^*	3.36	[1.30, 6.21]	2.24	[1.63,2.84]
r^*	1.26	[0.55, 2.10]	1.17	[0.56,1.90]
κ	0.73	[0.40, 1.16]	0.75	[0.39,1.22]
τ^{-1}	1.69	[1.02 2.50]	1.83	[1.09,2.72]
ρ_g	0.60	[0.45, 0.73]	0.79	[0.70,0.86]
ρ_z	0.81	[0.70, 0.90]	0.62	[0.46,0.78]
ρ_{gz}	-0.27	[-0.72, 0.25]	0.64	[0.23,0.92]
$M_{R\zeta}$	-0.16	[-1.51, 1.40]		
$M_{g\zeta}$	-1.91	[-2.80, -1.01]		
$M_{z\zeta}$	0.43	[0.09, 0.81]		
σ_R	0.15	[0.12, 0.20]	0.16	[0.12,0.21]
σ_g	0.26	[0.17, 0.38]	0.19	[0.14,0.27]
σ_z	0.69	[0.50, 0.94]	0.70	[0.51,0.96]
σ_ζ	0.19	[0.12, 0.28]		

Notes: The table reports posterior means and 90-percent probability intervals of the model parameters. The posterior summary statistics are calculated from the output of the Metropolis Hastings algorithm.

Table A3 - Prior and posteriors for structural parameters.

Name	Posterior Mean [5th pct, 95th pct] Stand. TR Indeterminacy	Stand. TR Determinacy	Alt. TR Indeterminacy	Alt. TR Determinacy
ψ_π	0.94 [0.83,1.00]	2.17 [1.46,3.03]	0.95 [0.83,1.04]	1.97 [1.31,2.83]
ψ_y	0.26 [0.07,0.55]	0.28 [0.07,0.59]	0.28 [0.07,0.60]	0.25 [0.06,0.55]
ψ_{gy}			0.36 [0.11,0.67]	0.42 [0.12,0.81]
ρ_R	0.73 [0.64,0.81]	0.79 [0.66,0.87]		
ρ_{R1}			1.11 [0.87,1.35]	1.20 [0.95,1.43]
ρ_{R2}			−0.37 [−0.58,−0.15]	−0.40 [−0.62,−0.17]
π^*	2.13 [1.01,3.49]	2.19 [1.58,2.81]	2.09 [1.02,3.43]	2.23 [1.57,2.91]
r^*	1.11 [0.51,1.79]	1.06 [0.51,1.70]	1.05 [0.49,1.69]	1.03 [0.49,1.66]
γ	0.49 [0.45,0.53]	0.55 [0.48,0.63]	0.51 [0.46,0.59]	0.55 [0.48,0.63]
θ	0.30 [0.22,0.39]	0.49 [0.36,0.61]	0.40 [0.26,0.62]	0.50 [0.38,0.62]
ρ_d	0.69 [0.53,0.84]	0.84 [0.76,0.92]	0.75 [0.58,0.89]	0.84 [0.75,0.91]
ρ_z	0.70 [0.59,0.81]	0.78 [0.64,0.89]	0.70 [0.56,0.83]	0.76 [0.61,0.88]
$M_{R\zeta}$	−0.29 [−1.55,1.06]		−0.43 [−1.90,1.02]	
$M_{d\zeta}$	−1.45 [−2.79,0.30]		−0.89 [−2.66,0.96]	
$M_{z\zeta}$	2.34 [1.62,3.14]		1.78 [−0.17,2.85]	
σ_R	0.17 [0.13,0.24]	0.18 [0.13,0.27]	0.17 [0.12,0.24]	0.17 [0.11,0.25]
σ_d	0.30 [0.14,0.59]	0.73 [0.48,1.08]	0.28 [0.14,0.54]	0.65 [0.40,0.99]
σ_z	0.46 [0.35,0.59]	0.58 [0.40,0.85]	0.55 [0.37,0.90]	0.61 [0.41,0.92]
σ_ζ	0.30 [0.16,0.53]		0.34 [0.16,0.65]	

Notes: The table reports posterior means and 90-percent probability intervals of the model parameters.